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Subject: The MODIS Thermal Calibration Algorithm (Draft)
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Objective

This chapter develops a non-linear absolute scene radiance equation and algorithm for the MODIS thermal bands. Single and multi-scan techniques are set forth to calibrate the coefficients of this equation based on the two on-orbit thermal calibrators. DC restore algorithms are presented to convert the telemetered data into detector voltages. Algorithms to calibrate the coefficients during lunar and other events are developed. Correction algorithms are developed to account for variations in the scan mirror reflectivity with respect to angle and side. An analysis of the sensitivity of the scene radiance equation to errors permutated in its parameters is then examined.

Methodology

The MODIS thermal algorithm utilizes two on-orbit calibrators in conjunction with three pre-launch calibratiors to determine the radiance emitted by the Earth. The on-orbit calibrators are the space view and the blackbody. This algorithm determines a non-linear scene radiance equation whose coefficients are calibrated based on these two calibrators every scan. The instrument contains 18 thermal bands, each band constituting a specific wavelength range. These bands are numbered 20-25, 27-36, 31hi, and 32hi.

The methodology described here to calibrate the thermal bands is divided into 12 sections:

1) depicts techniques for determining the radiance and the voltage of the on-orbit blackbody to be used in the determination of the calibration coefficients.

2) determines the radiance and voltage of the on-orbit space view to be used in the determination of the calibration coefficients

3) develops the non-linear scene radiance equation

4) calibrates the scene radiance equation of section 3 based on a single scan of blackbody and space view data.

5) calibrates the scene radiance equation of section 3 based on two scans of blackbody and space view data.

6) converts the telemetry data into a detector voltage.

7) examines calibration when the moon comes in through the space view.

8) examines calibration when the solar diffuser is lit by the sun.

9) examines calibration when the SRCA is activated.

10) examines techniques for accounting for the angular dependency of the scan mirror reflectivity.

11) examines techniques for accounting for the side dependency of the scan mirror reflectivity.

12) determines the non-linear coefficient based on pre-launch measurements.

1. DETERMINATION OF THE ON-ORBIT BLACKBODY RADIANCE AND VOLTAGE. The MODIS on-orbit blackbody is an anodized V-groove structure containing twelve thermistors. SBRC guarantees that the emissivity shall be greater than .992 and will be known to within .004. The blackbody temperature shall be known to within .1K.

As the MODIS focal plane assembly scans the blackbody, each detector scans 50 frames of the blackbody. These frames do not occur simultaneously since the detectors span an angle equivalent to 30 frames. To reduce the crosstalk and ghosting caused by the cavity walls, only those frames which occur when all the detectors simultaneously view the blackbody, will be used. Therefore, the center 20 frames, have a "clear view" of the blackbody. Furthermore, the two frames at either end will not be used to avoid viewing the blackbody mounting brackets. Hence, 16 frames of the blackbody will be viewed resulting in 15 integrated packets of data for each detector. The 15 blackbody voltages corresponding to these data packets will be averaged to obtain one voltage value for the blackbody (Vbb).

The 12 thermisters readings will also be averaged to obtain one temperature for the blackbody (Tbb). With a knowledge of the expected temperature gradients of the blackbody, an algorithm will be developed to identify faulty thermistors. Any thermistors which sufficiently deviate from the mean will not be used in the determination of the average blackbody temperature.

The radiance of the blackbody incident on the scan mirror can then be determined every scan by the following equation:

$$L_{\lambda bb} = \varepsilon_{bb} B_{\lambda bb} + \frac{1-\varepsilon_{bb}}{\pi}\left(\Omega_{cav} B_{\lambda cav} + \Omega_{Earth} B_{\lambda Earth}\right) \tag{1.1}$$

where:

ε_{bb} = blackbody emissivity (measured pre-launch)
Ω_{cav} = solid angle of the cavity subtended by the blackbody
Ω_{Earth} = solid angle of the Earth subtended by the blackbody

Since the desired units of radiance is Watts per meter squared per micron per steradian, the Planck function is:

$$B_{\lambda} = \frac{2hc^2}{\pi\lambda^5\left(e^{(ch/\lambda kT)} - 1\right)} \tag{1.2}$$

where:

$h = 6.6256 \pm 0.0005 \times 10^{-34}$ W sec^2
$c = 2.997925 \pm .000003 \times 10^{8}$ m / sec
$k = 1.38054 \pm 0.00018 \times 10^{-23}$ W sec / K
λ = wavelength in μm
T = temperature in Kelvin of the blackbody, cavity or Earth

Due to the spectral width of the band pass filters, the blackbody radiance could be more accurately determined by integrating across the normalized spectral response (R_{band}) of the filter. The blackbody radiance equation then becomes:

$$L_{bb(i)} = \sum_{\lambda lower}^{\lambda upper} L_{\lambda bb} R_{band} \Delta\lambda \tag{1.3}$$

where:

λ_{upper} Upper wavelength limit of the filter (determined pre-launch)

λ_{lower} Lower wavelength limit of the filter (determined pre-launch)

It is not feasible to perform this integral every scan. The preferred method for evaluating this integral will be to create band dependent tables of radiance values for a discrete set of temperature values. Two tables are neccessary for each band. One table for the emitted radiance attributed to the blackbody itself, the other for the radiance attributed to an external radiance source reflected from the blackbody. The temperature range of the emitted radiance table will be from 280K to 320K with a resolution of .05K. The temperature range of the reflected radiance table will be from 250K to 350K with a resolution of .1K. Utilizing these look-up tables, the blackbody radiance equation now becomes:

$$L_{bb} = L_{em(Tbb)} + \frac{1}{\pi}\left(\Omega_{cav} L_{ref(Tcav)} + \Omega_{Earth} L_{ref(TEarth)}\right) \tag{1.4}$$

where the emitted blackbody radiance table can be determined by:

$$L_{em(T)} = \sum_{\lambda lower}^{\lambda upper} \varepsilon_{bb} B_{\lambda(T)} R_{band} \Delta\lambda \tag{1.5}$$

and the reflected blackbody radiance table can be determined by:

$$L_{ref(T)} = \sum_{\lambda lower}^{\lambda upper} (1-\varepsilon_{bb}) B_{\lambda(T)} R_{band} \Delta\lambda \tag{1.6}$$

2. DETERMINATION OF THE ON-ORBIT SPACE VIEW RADIANCE AND VOLTAGE. The MODIS on-orbit space view is a hole in the MODIS cavity. As the MODIS focal plane assembly scans the space view, each detector scans 50 frames of the space view. These frames do not occur simultaneously since the detectors span an angle equivalent to 30 frames. To reduce the crosstalk and ghosting caused by the cavity walls, only those frames which occur when all the detectors simultaneously scan the space view, will be used. Therefore, the center 20 frames, have a "clear view" of space. Furthermore, the two frames at either end will not be used to avoid viewing the cavity wall edges. Hence, 16 frames of the space view will be scanned resulting in 15 integrated packets of data for each detector. The 15 space view voltages corresponding to these data packets will be averaged to obtain one voltage value for the space view (Vsv). The radiance of the space view incident on the scan mirror will be assumed to the zero.

3. THE THERMAL SCENE RADIANCE EQUATIONS. The thermal scene radiance equation can be determined by the relationship between the detector voltage and the radiance incident at the aperture. The aperture will be defined as the surface of the scan mirror. In the form of a quadratic, this relationship can be expressed in one of two ways. The scene radiance equation can be expressed as:

$$L_{ap} = j(V_{det}-V_o)^2 + k(V_{det}-V_o) - L_o \qquad 3.1$$

where:

L_{ap}	Radiance incident at the aperture (scan mirror)
V_{det}	Unamplified detector voltage
V_o	Pre-flight measured detector voltage for zero irradiance at the detector.
L_o	At-aperture equivalent internal optical background radiance term
k	First order term gain coefficient
j	Second order term gain coefficient

The equation can also be expressed as:

$$V_{det} = q(L_{ap}+L_o)^2 + m(L_{ap}+L_o) + V_o \qquad 3.2$$

where:

m	First order term gain coefficient
q	Second order term gain coefficient

Since the measured variable is Vdet, equation 3.1 is of the more desirable form. However, Tom Pagano of SBRC claims the form of equation 3.2 will more accurately fit real detector data. Therefore, equation 3.2 will be used to determine the scene radiance.

The unknown variables in equation 3.2 are L_o, m, and q. L_o is the effective radiance at the MODIS aperture due to the internal optical background. The internal optical background radiance is not the cavity background radiance, but is the radiance produced within the optics. L_o does not actually occur at the MODIS aperture. It is a radiance that, if applied to the aperture of the optical system with no internal background, would create a detector irradiance equivalent to that of the optical system with the internal background. It is effectively a virtual form of the internal optical background. Direct measurement of the individual parameters contributing to the internal optical background of the system is not necessary, since these parameters are contained within the unknown variables L_o, m, and q (see error analysis).

These equations assume the detector circuit to be designed such that V_{det} is the voltage across the load resistor for the photoconductive (PC) bands and the voltage across the detector for the photovoltaic (PV) bands. This is essential since we desire the detector voltage to increase as the scan mirror incident radiance increases for a constant internal optical background. If the circuit design is contrary to this, then the detector voltage must be subtracted from the bias voltage to obtain the appropriate V_{det} to be used in the thermal scene radiance equations.

4. UTILIZATION OF THE ON-ORBIT BLACKBODY AND THE ON-ORBIT SPACE VIEW TO CALIBRATE THE SCENE RADIANCE EQUATION. Equation 3.2 can be applied to the two on-orbit calibrators every scan. Since this yields two equations with three unknowns, we are forced to determine one unknown pre-launch. Because the value of q will vary less than the values of m and L_o, q will be determined pre-launch as a constant q_{av} (see section 12). The scene radiance equation now reduces to an equation with two unknowns, L_o and m.

$$V_{det} = V_o + m(L_{sp}+L_o) + q_{av}(L_{sp}+L_o)^2 \qquad 4.1$$

The average detector voltage obtained when scanning the blackbody and the calculated radiance of the blackbody can be applied to equation 4.1 as follows:

$$V_{bb} = V_o + m(L_{bb}+L_o) + q_{av}(L_{bb}+L_o)^2 \qquad 4.2$$

where:

L_{bb}	Radiance of the on-orbit blackbody incident on the scan mirror
V_{bb}	Detector voltage when viewing the on-orbit blackbody

The average detector voltage obtained when scanning the space view can be applied to equation 4.1 as follows:

$$V_{sv} = V_o + m\left(\rho_{sv/bb}L_{sv}+L_o\right) + q_{av}\left(\rho_{sv/bb}L_{sv}+L_o\right)^2 \qquad 4.3$$

where:

L_{sv}	Radiance of the on-orbit space view incident on the scan mirror
V_{sv}	Detector voltage when viewing the on-orbit space view

The quantity $\rho_{sv/bb}$ is the scan mirror reflectivity during the space view scan relative to the scan mirror reflectivity during the blackbody scan. This relative reflectivity term is due to angular variations in scan mirror reflectivity. Since the radiance of the on-orbit space view is effectively zero, equation 4.3 reduces to:

$$V_{sv} = V_o + mL_o + q_{av}L_o^2 \qquad 4.4$$

Combining the equations 4.2 and 4.4, Lo can be expressed as:

$$L_o = \frac{V_{bb} - V_{sv} - q_{sv}L_{bb}^2 \pm \sqrt{(V_{sv} - V_{bb} + q_{sv}L_{bb}^2)^2 + 4q_{sv}L_{bb}^2(V_o - V_{sv})}}{2q_{sv}L_{bb}} \qquad 4.5$$

where the correct root is the negative root.

Solving equation 4.4 for m, yields:

$$m = \frac{V_{sv} - V_o}{L_o} - q_{sv}L_o \qquad 4.6$$

The generalized scene radiance equation can now be expressed specifically for the Earth view scene:

$$V_{ev} = V_o + m\left(\rho_{ev/bb(\theta ev)}L_{ev} + L_o\right) + q_{sv}\left(\rho_{ev/bb(\theta ev)}L_{ev} + L_o\right)^2 \qquad 4.7$$

where:

L_{ev}	Radiance of the Earth view scene incident on the scan mirror
V_{ev}	Detector voltage when viewing the Earth view scene

The quantity $\rho_{ev/bb(\theta ev)}$ is the scan mirror reflectivity during the Earth view scan relative to the scan mirror reflectivity during the blackbody scan. This quantity is equal to either $\rho_{Aev/bb(\theta ev)}$ or $\rho_{Bev/bb(\theta ev)}$, depending on the mirror side (see section 10).

To express L_{ev} as a function of V_{ev}, it is nesessary to apply the quadratic formula. In so doing, we must require that q not be zero. If q is zero, there is no need to apply the quadratic. The calibrated scene radiance equation now becomes:

$$L_{ev} = \frac{-m - 2q_{sv}L_o \pm \sqrt{m^2 + 4q_{sv}(V_{ev} - V_o)}}{2q_{sv}\rho_{ev/bb(\theta ev)}} \qquad 4.8$$

where the correct root is the positive root.

5. MULTI-SCAN CALIBRATION OF THE SCENE RADIANCE EQUATION. To help correct noise variations between the blackbody, space view, and Earth view frames of data, a mult-scan interpolation of the calibration parameters can be useful. Adapting equation 4.5 to a two-consecutive-scan interpolation where the Earth view scan is the first (i^{th}) scan, yields:

$$L_{o(i)} = \frac{V_{bb\,int(i)} - V_{sv\,int(i)} - q_{sv}L^2_{bb\,int(i)} \pm \sqrt{(V_{sv\,int(i)} - V_{bb\,int(i)} + q_{sv}L^2_{bb\,int(i)})^2 + 4q_{sv}L^2_{bb\,int(i)}(V_o - V_{sv\,int(i)})}}{2q_{sv}L_{bb\,int(i)}} \qquad 5.1$$

The voltage of the blackbody interpolated to a specific Earth view scan angle is:

$$V_{bb\,int(i)} = w_{bb1} V_{bb(i)} + w_{bb2} V_{bb(i+1)} \qquad 5.2$$

where the blackbody weighting factors are:

$$w_{bb1} = \frac{\theta_{bb} - \theta_{ev}}{360} \qquad 5.3$$

$$w_{bb2} = \frac{360 - \theta_{bb} + \theta_{ev}}{360} \qquad 5.4$$

The scan angle (θ) is in units of degress with nadir equal to zero. These angles are band dependent due the angular offset of the bands. The angular range of the optical axis is: 230.75 degrees to 232.05 degrees for the blackbody, 260.9 degrees to 262.25 degrees for the space view, and -55 degrees to 55 degrees for the Earth view. The voltage of the space view interpolated to a specific Earth view scan angle is:

$$V_{sv\,int(i)} = w_{sv1} V_{sv(i)} + w_{sv2} V_{sv(i+1)} \qquad 5.5$$

where the space view weighting factors are:

$$w_{sv1} = \frac{\theta_{sv} - \theta_{ev}}{360} \qquad 5.6$$

$$w_{sv2} = \frac{360 - \theta_{sv} + \theta_{ev}}{360} \qquad 5.7$$

The reflectivity of side A of the scan mirror will differ from that of side B. Since an interpolation of the blackbody radiances involves both side A and side B, it is neccessary to account for this reflectivity variation. The interpolated blackbody radiance integrated across the normalized spectral response of the filter is:

$$L_{bb\,int(i)} = w_{bb1} \sum_{\lambda lower}^{\lambda upper} L_{\lambda bb(i)} R_{(i)} \Delta\lambda + w_{bb2} \frac{\rho_{(i+1)}}{\rho_{(i)}} \sum_{\lambda lower}^{\lambda upper} L_{\lambda bb(i+1)} R_{(i+1)} \Delta\lambda \qquad 5.8$$

The relative reflectivity ratio is band dependent and is determined as a constant across the bandwidth for each band

Adapting equation 4.6 to the two-scan data set yields:

$$m_{(i)} = \frac{V_{sv\,int(i)} - V_o}{L_{o(i)}} - q_{sv}L_{o(i)} \qquad 5.9$$

6. DETECTOR VOLTAGE DETERMINATION FROM MODIS DIGITAL NUMBER, DC RESTORE VOLTAGE AND ELECTRONIC GAIN. In order to preserve the dynamic range of the 12 bit A/D converter, electronic gains and DC restore voltages are applied to both the PC and PV circuits for every scan. Since the thermal calibration equation requires the voltage at the detector, we must develop an equation which converts this combination of electronic gains, DC restore voltages, and MODIS digital numbers (DN) into a detector voltage. The electonic gains and DC restore offset voltages are applied differently for the PV than the PC circuits.

The detector voltage of the PV bands, which contain only one gain and one DC restore voltage source can be determined as follows:

$$V_{s(i)} = \frac{DN_{s(i)} - DN_o}{G_{(i)}R_{A/D}} - V_{DC(i)} \qquad 6.1$$

where:

$R_{A/D}$	Responsivity of the A/D converter
DN_s	MODIS digital number output of the 12 bit A/D converter
DN_o	MODIS digital number offset of the 12 bit A/D converter ($DN_o = 100$)
G	Electronic gain of the PV circuit
V_{DC}	DC restore voltage of the PV circuit applied by the AEM

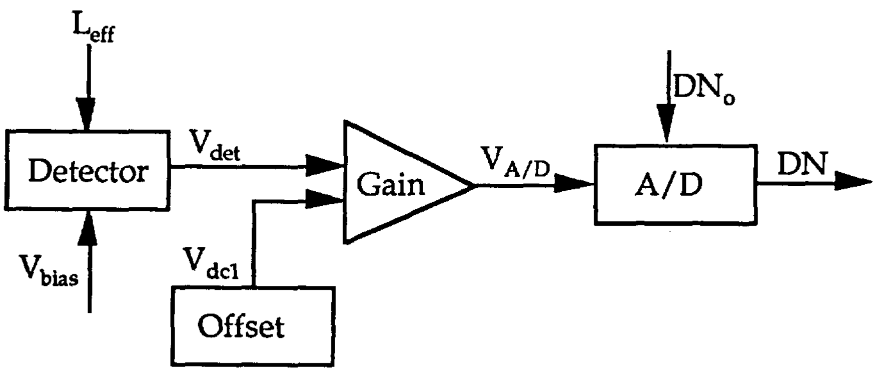

Fig. 1 Photovoltaic DC Restore Circuit (Bands 1 - 30)

The detector voltage of the PC bands, which contain two gains and two DC restore voltage sources, can be determined as follows:

$$V_{s(i)} = \frac{DN_{s(i)} - DN_o}{G_{1(i)} G_{2(i)} R_{A/D}} - \frac{V_{DC2(i)}}{G_{1(i)}} - V_{DC1(i)} \qquad 6.2$$

where:

G1	First electronic gain of the PC circuit
G2	Second electronic gain of the PC circuit
V_{DC1}	First DC restore voltage of the PC circuit applied by the AEM
V_{DC2}	Second DC restore voltage of the PC circuit applied by the AEM

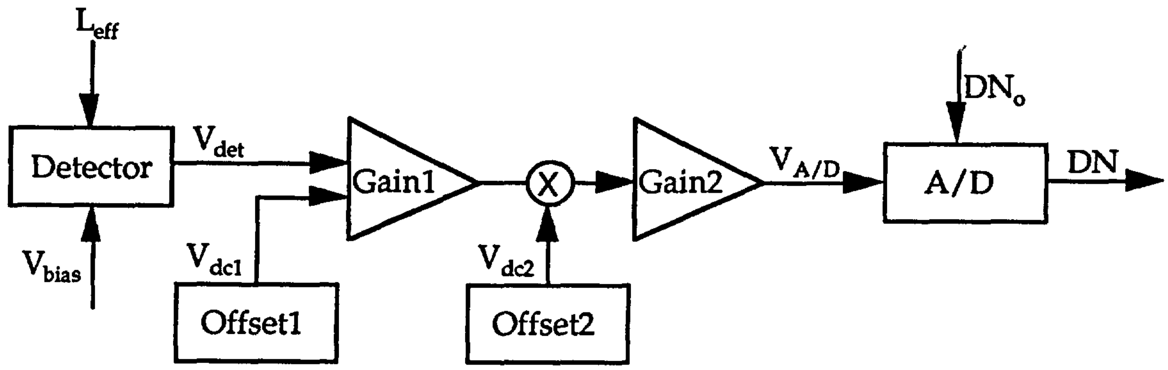

Fig. 2 Photoconductive DC Restore Circuit (Bands 31 - 36)

The responsivity of the A/D converter will vary on-orbit and can be determined as follows:

$$R_{A/D} = \frac{2^{12}}{V_{A/Dmax}} \qquad 6.3$$

where:

$V_{A/Dmax}$ Full scale voltage of the A/D converter

7. CALIBRATION OF THE SCENE RADIANCE EQUATION DURING LUNAR EVENTS. The algorithm for determining the value of the calibration coefficients during lunar exposure of the space view is dependent on the number of scans of data available. The duration of the lunar events will be approximately 7 scans. If the first scan of lunar exposure is scan j and the last scan of lunar exposure is scan k, then the coefficients for scans j - 2 through k + 2 will be determined in this section. Scan j - 2 will be considered the last "good" space view scan and scan k + 2 will be considered the first "good" space view scan.

Two algorithms are presented here. The first utilizes a 2 scan data set and the second utilizes an N scan data set. The value of N will be no greater than 12, where:

$$N = k - j + 5 \tag{7.1}$$

where:

N   Number of scans in data set
j   first scan of lunar event
k   last scan of lunar event

The determination of which scan is the first or last scan of the lunar event will be made vicariously. A backup multi-scan algorithm could be developed which looks for any sufficient rise or fall of the space view detector voltage to determine the beginning and end points.

7-A. Algorithm to Freeze Coefficients. With the limitations of a 2 scan data set, the best calibration option is to freeze the coefficients of scan j - 2.

$$m_{(i)} = m_{(j-2)} \tag{7.2}$$

$$L_{o(i)} = L_{o(j-2)} \tag{7.3}$$

7-B. Algorithm to Interpolate Coefficients. This algorithm requires an N scan data set and is more accurate than 7-A since it can account for any steady optical background change caused by the lunar event. The space view voltage, blackbody voltage, and blackbody radiance values used to calibrate the coefficients are interpolated between scans j - 2 and K + 2.

The space view voltage equation becomes:

$$V_{sv\,int(i)} = w_{lunar1} w_{sv1} V_{sv(j-2)} + w_{lunar2} w_{sv2} V_{sv(k+2)} \tag{7.4}$$

The blackbody voltage becomes:

$$V_{bb\,int(i)} = w_{lunar1} w_{bb1} V_{bb(j-2)} + w_{lunar2} w_{bb2} V_{bb(k+2)} \tag{7.5}$$

The blackbody radiance becomes:

$$L_{bb\,int(i)} = w_{lunar1} w_{bb1} \sum_{\lambda lower}^{\lambda upper} L_{\lambda bb(i)} R_{(i)} \Delta\lambda + w_{lunar2} w_{bb2} \frac{\rho_{(i+1)}}{\rho_{(i)}} \sum_{\lambda lower}^{\lambda upper} L_{\lambda bb(i+1)} R_{(i+1)} \Delta\lambda \qquad 7.6$$

where the calibration weights are:

$$w_{lunar1} = \frac{(k-i+2)}{N-1} \qquad 7.7$$

$$w_{lunar2} = \frac{(i-j+2)}{N-1} \qquad 7.8$$

The values obtained from equations 7.4 through 7.6 are then used as input to equations 5.1 and 5.9 to determine coefficients.

8. CALIBRATION OF THE SCENE RADIANCE EQUATION DURING SOLAR DIFFUSER EVENTS.

TBD

9. CALIBRATION OF THE SCENE RADIANCE EQUATION DURING SRCA EVENTS.

TBD

10. ON-ORBIT DETERMINATION OF THE SCAN MIRROR ANGLE DEPENDENT RELATIVE RELATIVITY. The calculated radiance for all the Earth view frames of data should average out to one value after a sufficient number of scans. Variations in these calculated radiance values can be used to determine the relative reflectivity of the scan mirror at any Earth view angle with respect to the reflectivity of the scan mirror at an Earth view frame equivalent to the blackbody view angle.

The average calibrated value of the Earth view radiance for each scan angle can be computed as follows:

$$\bar{L}_{Aev(\theta ev)} = \frac{1}{N} \sum_{i=1}^{N} L_{Aev(\theta ev)(1000i)} \qquad 10.1$$

This equation assumes that scan i corresponds to mirror side A. To avoid redundant data due to scene similaries in neighboring scans and to insure the exclusive use of side A of the scan mirror, a reasonable scan incrementation of one

Earth view radiance calculation every 1000 scans will be effected by equation 10.1. N is the total number of scans necessary to obtain a stabilized average of the Earth view radiance. The equation can be applied to side B of the mirror as:

$$\bar{L}_{Bev(\theta ev)} = \frac{1}{N}\sum_{i=1}^{N} L_{Bev(\theta ev)(1000i+1)} \qquad 10.2$$

The relative reflectivity of side A of the scan mirror at any Earth view angle with respect to the blackbody angle is then:

$$\rho_{Aev/bb(\theta ev)} = \frac{\bar{L}_{Aev(\theta ev)}}{\bar{L}_{Aev(\theta ev=\theta bb)}} \qquad 10.3$$

and side B is:

$$\rho_{Bev/bb(\theta ev)} = \frac{\bar{L}_{Bev(\theta ev)}}{\bar{L}_{Bev(\theta ev=\theta bb)}} \qquad 10.4$$

11. ON-ORBIT DETERMINATION OF THE SCAN MIRROR SIDE DEPENDENT RELATIVE RELATIVITY. The scan mirror side dependent relative reflectivity is only neccessary if the calibration utilizes the multi-scan interpolation technique. In this case, the second scan utilizes the opposite side of the scan mirror to measure the blackbody and space view. Assuming the radiance from the space view to be zero, it is only neccessary to calculate the side dependent relative reflectivity at the blackbody view angle. The following calculations will be done for each detector.

The relative reflectivity ($\rho_{B/A}$) of mirror side B with respect to mirror side A will be calculated based on the data taken from two consecutive scans. The input parameters are:

$L_{bb(A)}$	Mirror side A blackbody radiance (see section 1)
$L_{bb(B)}$	Mirror side B blackbody radiance (see section 1)
$V_{bb(A)}$	Mirror side A blackbody detector voltage (see section 6)
$V_{bb(B)}$	Mirror side B blackbody detector voltage (see section 6)
$V_{sv(A)}$	Mirror side A space view detector voltage (see section 6)

The calibration equation coefficients will be determined based on scan mirror side A data. Equation 4.5 now becomes:

$$L_o = \frac{V_{Abb} - V_{Asv} - q_{av}L^2_{Abb} \pm \sqrt{(V_{Asv} - V_{Abb} + q_{av}L^2_{Abb})^2 + 4q_{av}L^2_{Abb}(V_o - V_{Asv})}}{2q_{av}L_{Abb}} \qquad 11.1$$

and equation 4.6 now becomes:

$$m = \frac{V_{Asv} - V_o}{L_o} - q_{av} L_o \qquad 11.2$$

The radiance of the blackbody at side B will now be determined by applying the scene radiance equation 4.8 to the blackbody detector voltage at side B:

$$L_{Bbbcal} = \frac{-m - 2q_{av}L_o \pm \sqrt{m^2 + 4q_{av}(V_{Bbb} - V_o)}}{2q_{av}} \qquad 11.3$$

Since the noise difference between scan mirror sides will average to zero over a large number of scans, $\rho_{B/A}$will be determined over a large number of scans. The side dependent relative reflectivity ratio can then be determined by:

$$\rho_{B/A} = \sum_{i=1}^{N} \frac{L_{bbcal(B)}}{L_{bb(B)}} \qquad 11.4$$

Once a stabilized average $\rho_{B/A}$ is obtained, it will not be neccessary to continuously calculate this quantity. This value can be checked periodically and adjusted off-line if a change is detected.

12. DETERMINATION OF THE PRE-LAUNCH SECOND ORDER TERM GAIN COEFFICIENT. The non-linear variable q will be determined pre-launch by applying equation 3.2 to the three pre-launch calibrators: the blackbody, the space view, and the ground-based calibrator. This yields three equations with three unknowns: m, L_o, and q. Combining these three equations and solving for q gives:

$$q = \frac{(V_{gb} - V_o)(L_{bb} + L_o) - (V_{bb} - V_o)\left(\rho_{gb/bb}L_{gb} + L_o\right)}{(L_{bb} + L_o)\left(\rho_{gb/bb}L_{gb} + L_o\right)^2 - \left(\rho_{gb/bb}L_{gb} + L_o\right)(L_{bb} + L_o)^2} \qquad 12.1$$

where:

$$L_o = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad 12.2$$

$$A = \rho_{gb/bb}L_{gb}(V_{sb} - V_{sv}) + L_{bb}(V_{sv} - V_{gv}) \qquad 12.3$$

$$B = \rho^2_{gb/bb}L^2_{gb}(V_{sb} - V_{sv}) + L^2_{bb}(V_{sv} - V_{gb}) \qquad 12.4$$

$$C = \rho_{gb/bb}(V_o - V_{sv})\left(\rho_{gb/bb}L_{bb}L_{gb}^2 - L_{gb}L_{bb}^2\right) \qquad 12.5$$

L_o	Pre-launch at-aperture equivalent background radiance term
q	Second order term gain coefficient
q_{av}	Average second order term gain coefficient (average of q)
L_{bb}	Radiance of the pre-launch blackbody incident on the scan mirror
V_{bb}	Voltage out of the fpa when viewing the on-orbit blackbody
L_{sv}	Radiance of the pre-launch space view incident on the scan mirror
V_{bb}	Voltage out of the fpa when viewing the on-orbit blackbody
L_{gb}	Radiance of the pre-launch ground-based calibration source incident on the scan mirror
V_{gb}	Voltage out of the fpa when viewing the pre-launch ground-based calibration source

The on-orbit non-linear thermal calibration constant (q_{av}) will be then determined as the average of the pre-launch q values.

$$q_{av} = \frac{1}{N}\sum_{i=1}^{N} q_{(i)} \qquad 12.6$$

where:

i	pre-flight scan number

Error Analysis and Budget

TBD

Verification

Verification of the thermal calibration algorithm can be done using the data generated during the thermal vacuum tests. The non-linear constant will be determined as described earlier. The pre-launch blackbody and the pre-launch space view data will then be treated as on-orbit data input to equation 8 for every pre-launch scan. This will not account for any long term variations in the instrument and detector, but will provide a basis for analyzing the effect of treating the non-linear variable as a constant.

Schedule
TBD

Risk Assessment

Their is concern the that zero radiance voltage off the detector (Vo) could change during the lifetime of the detector. A change in temperature of the detector would generate errors.

Appendix A: Derivation of Equations
TBD

Appendix B: Error Analysis Methodology
TBD

Appendix C: Thermal Band Characteristics
TBD

Nomenclature
TBD

References
TBD