A stochastic radiative transfer model of a discontinuous vegetation canopy

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ABSTRACT

From the viewpoint of interaction with electromagnetic radiation, a vegetation canopy can be considered to be a stochastically macro-nonhomogeneous scattering medium characterized by a random geometric structure (random shapes, dimensions, amounts and spatial locations of individual plant crowns) and a stochastic internal structure, such as the macroscale fluctuations in optical properties and dimensions of leaves. To model this interaction, the most promising approach is the use of a stochastic radiative transfer equation, whose coefficients are random scalar fields of the optical parameters of the elementary volumes of plant crowns. By averaging the stochastic transfer equation over the ensemble of realizations of the stochastic canopy field, we can thus obtain the lower-order moments of the stochastic radiation field, such as its mean and variance.

In this work, we formulate the radiative transfer equation for the ensemble average radiance based on a two-component random mixture model from kinetic theory. The resulting model can account for non-Markovian statistics as well as both vertical and lateral variations in the canopy. The key parameters of this model include the ratio of the height of the plant crown to its horizontal dimension and the percentage coverage of crowns on the ground. In addition are parameters of the ordinary one-dimensional canopy radiative transfer model. The radiative transfer equation for this model is solved most accurately using the Gauss-Seidel iteration algorithm; the asymptotic solution is comparable to that of the deterministic model we have developed earlier.

INTRODUCTION

There are many versatile techniques for solving the radiative transfer equation in a plane parallel canopies, (Goel, 1988; Myneni et al., 1990), which assume that the canopy is horizontally homogeneous and infinite. However, we frequently observe three dimensional isolated canopies or canopy fields composed of three dimensional plant crowns on the earths surface. Myneni et al. (Myneni, et al., 1990) developed a three-dimensional radiative transfer model of leaf canopy, that is horizontally inhomogeneous and infinite. Their numerical calculations (Myneni, et al., 1992) show that radiative transfer in the inhomogeneous canopy is rather different from its plane parallel counterpart. Suits (1983) proposed a technique to investigate radiative transfer in the discontinuous canopy by use of a so-called modulation function. This technique has been employed with the SAIL model to investigate of radiative transfer of crop canopies by Goel and Grier (Goel and Grier, 1986; 1988). Given a realization of of the canopy field, these models can provide us the spatial and angular characteristics of the canopy radiation field. Since the spatial distribution of the canopies are random in nature, the calculation of the statistical characteristics of the canopy radiation field will require a number of runs of computer codes based on different realizations of the canopies, which are very computationally expensive.

In many cases, canopies are stochastically macro-nonhomogeneous scattering media as a result of the random geometric structure (random shapes, dimensions, amounts and spatial locations of individual canopy) and stochastic internal structure, such as the macroscale fluctuations in optical properties and dimensions of leaves. This paper will deal with the canopies with random geometric but deterministic internal structure.

It is obvious that the radiation field of a stochastic canopy must be a random field. The most promising approach is the use of a stochastic transfer equation, whose coefficients are random scalar fields of the optical parameters of the elementary volumes of canopies. By averaging the stochastic transfer equation over the ensemble of realizations of the stochastic canopy field, it makes us possible to obtain the lower-order moments of the stochastic radiation field. This approach has been used for the broken cloud (Titiv, 1990; Zuev et al, 1987). The similar idea already has been used for canopies (Anisimov and Menzhulin, 1981).

THE MODEL OF THE CANOPY RANDOM FIELD

Let a canopy field occupies the layer Λ: $0 \le z \le H$ in the OXYZ cartesian coordinate system. The coefficients of extinction $\sigma(\mathbf{r}, \omega)$ and scattering $\sigma_s(\mathbf{r}, \omega)$ are the random scalar fields, namely, $\sigma(\mathbf{r}, \omega) = \sigma(\omega)\, k(\mathbf{r})$ and $\sigma_s(\mathbf{r}, \omega) = \sigma_s(\omega)\, k(\mathbf{r})$, where $k(\mathbf{r})$ is the random indicator function, i.e.

$$k(\mathbf{r}) = \begin{cases} 1 & \text{if } \mathbf{r} \in G \\ 0 & \text{if } \mathbf{r} \notin G \, , \end{cases} \tag{1}$$

where G is the random set of points in Λ covered with canopies. The field $k(\mathbf{r})$ is assumed the poison point field. The statistical characteristics of optical parameters within Λ are completely determined by the probabilistic properties of a random field $k(\mathbf{r})$. Based on some simple assumption, the first two moments are (Titov, 1990)

$$\begin{aligned} &< k(\mathbf{r}) > = P \\ &< k(\mathbf{r}_1)\, k(\mathbf{r}_2) > = P\; V(\mathbf{r}_1, \mathbf{r}_2) \\ &V(\mathbf{r}_1, \mathbf{r}_2) = (1 - P) \exp[-A(\omega) |\mathbf{r}_1 - \mathbf{r}_2|] + P \; , \end{aligned} \tag{2}$$

where the angular brackets denote the expected value of the field $k(\mathbf{r})$, $A(\omega) = |a| A_x + |b| A_y + |c| A_z$, the direction vector ω can be calculated by $\omega = (\mathbf{r}_1 - \mathbf{r}_2)/|\mathbf{r}_1 - \mathbf{r}_2| = (a, b, c)$, and $V(\mathbf{r}_1, \mathbf{r}_2) = P \left\{ k(\mathbf{r}_1) = 1 \mid k(\mathbf{r}_2) = 1 \right\}$ is the conditional probability of the canopy occupied in the point $\mathbf{r}_1$, under condition that the point $\mathbf{r}_2$ is covered with canopies. A_x, A_y and A_z are the average numbers of points per unit length.

According to (2), the field $k(\mathbf{r})$ is statistically homogeneous and anisotropic and has the exponential correlation factor

$$B(\mathbf{r}_1 - \mathbf{r}_2) = B(|x_1 - x_2|)\, B(|y_1 - y_2|)\, B(z_1 - z_2|) \; .$$

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From the stochastic properties of the poison field, the absolute canopy coverage N can be expressed as:

$$N = 1-(1-P)\exp(-A_z H) \quad . \tag{3}$$

Since the canopy has the random altitude at the top and the bottom, $A_z \neq 0$ in general. From (3) we can see $N \geq P$.

RADIATIVE TRANSFER MODEL

For a random leaf canopy, the radiative transfer equation can be written as:

$$\omega \nabla I(\mathbf{r}, \omega) + \sigma(\omega) k(\mathbf{r}) I(\mathbf{r}, \omega) = \int_{4\pi} \sigma_s(\omega' \to \omega)\, k(\mathbf{r})\, I(\mathbf{r}, \omega') d\omega' \tag{4}$$

where the total cross-section $\sigma(\omega)$ is defined as (Shultis and Myneni, 1988)

$$\sigma(\omega) = u_l\, G(\omega) \ ,$$

and u_l is the leaf area density and $G(\omega)$ is related to the leaf angle distribution function $g_l(\omega_l)$

$$G(\omega) = \frac{1}{2\pi}\int_{2\pi+} g_l(\omega_l) \,|\, \omega\, \omega_l \,|\, d\omega_l \ ,$$

where $2\pi+$ indicates the integration over the upper hemisphere. The functions u_l and $g_l(\omega_l)$ characterize the architecture of the leaf canopy. The differential scattering cross-section $\sigma_s(\omega' \to \omega)$ may be expressed by (Shultis and Myneni, 1988)

$$\sigma_s(\omega' \to \omega) = u_l\, \Gamma(\omega' \to \omega)/\pi \ ,$$

where $\Gamma(\omega' \to \omega)$ is the area scattering phase function originally introduced by Ross (1981).

In order to solve the radiative transfer equation effectively, the differential operator in equation (4) is changed to the integration operator, thus the radiative transfer equation can be expressed:

$$\begin{aligned} I(\mathbf{r}, \omega) &+ \frac{1}{|c|}\int_{E_z} u_l\, G(\omega) k(\mathbf{r}') I(\mathbf{r}', \omega) d\xi \\ &= \frac{u_l}{\pi |c|}\int_{E_z} \Gamma(\omega_0 > \omega) I(\mathbf{r}', \omega) k(\mathbf{r}') d\xi \end{aligned} \tag{5}$$

where

$$E_z = \begin{cases} (0, z), & c > 0 \\ (z, H), & c < 0 \end{cases}$$

Let's average above equations over the ensemble of the $k(\mathbf{r})$ random field realizations, then

$$\begin{aligned} \langle I(\mathbf{r}, \omega)\rangle &+ \frac{u_l P G(\omega)}{|c|}\int_{E_z} U(\mathbf{r}', \omega) d\xi \\ &= \frac{u_l P}{\pi |c|}\int_{E_z} \Gamma(\omega_0 \to \omega)\, U(\mathbf{r}', \omega) d\xi \end{aligned} \tag{6}$$

where U function is defined as

$$U(\mathbf{r}, \omega) = \langle k(r)\, I(\mathbf{r}, \omega)\rangle / P$$

If equations (5) is multiplied by $k(\mathbf{r})$ and averaged once again, we have

$$\begin{aligned} P\, U(\mathbf{r}, \omega) &+ \frac{u_l\, G(\omega)}{|c|}\int_{E_z} \langle k(\mathbf{r}) k(\mathbf{r}') I(\mathbf{r}', \omega)\rangle\, d\xi \\ &= \frac{u_l}{\pi |c|}\int_{E_z} \Gamma(\omega_0 \to \omega) \langle k(\mathbf{r})\, k(\mathbf{r}')\, I(\mathbf{r}', \omega)\rangle\, d\xi \end{aligned}$$

In order to generate the closure equations, we have to split the correlation function $\langle k(\mathbf{r})\, k(\mathbf{r}')\, I(\mathbf{r}', \omega)\rangle$. Based on the Markovian properties of the Poison process, one splitting formula has been used to approximate the correlation function (Titov, 1990)

$$\langle k(\mathbf{r})\, k(\mathbf{r}')\, I(\mathbf{r}', \omega)\rangle = V(\mathbf{r}, \mathbf{r}') \langle k(\mathbf{r}')\, I(\mathbf{r}', \omega)\rangle \ . \tag{7}$$

Thus above equation for the mean radiance can be described:

$$\begin{aligned} U(\mathbf{r}, \omega) &+ \frac{u_l\, G(\omega)}{|c|}\int_{E_z} V(\mathbf{r}, \mathbf{r}') d\xi \\ &= \frac{u_l}{\pi |c|}\int_{E_z} \Gamma(\omega_0 \to \omega) V(\mathbf{r}', \mathbf{r})\, U(\mathbf{r}', \omega) d\xi \end{aligned} \tag{8}$$

So far the closed equation systems for the mean radiance have been derived. As soon as $U(\mathbf{r}, \omega)$ is solved from (8), the mean radiance $\langle I(\mathbf{r}, \omega)\rangle$ can be calculated from (6). Note that since the canopy field is statistically homogeneous and the boundary conditions are homogeneous, then

$$\langle I(\mathbf{r}, \omega)\rangle = \langle I(z, \omega)\rangle$$

$$\langle U(\mathbf{r}, \omega)\rangle = \langle U(z, \omega)\rangle \ ,$$

which implies that we have converted a three-dimensional problem into a one-dimensional.

It is noticed that the models introduced in the kinetic theory literature concerning particle and radiation transport in stochastic media can very easily be applied to canopy radiative transfer. Malvagi et al. (1993) demonstrated that radiative transfer equation for the partially cloudy atmosphere given by Titov (1990) is the special case of the two-component mixture model. This special case is the case corresponding to no emission, no interaction between the radiation and the clear sky, and Markovian statistics. If we treat canopy as one component, and air between the canopy is another component, the two-component stochastic mixture theory can be used to account for arbitrary (non-Markovian) size and spacing distributions, and it has the form of integro-differential equations that are convenient for analysis.

Thus, the averaged mean radiance equation for the stochastic canopy field can be derived from two-component stochastic mixture theory

$$\omega \nabla (p_0 I_0) = \frac{p_1}{\lambda_1} I_1 - \frac{p_0}{\lambda_0} I_0$$

$$\omega \nabla (p_1 I_1) + \sigma_1 p_1 I_1 = \int_{4\pi} \sigma_s p_1\, I_1 d\omega - \frac{p_1}{\lambda_1} I_1 + \frac{p_0}{\lambda_0} I_0$$

where the average radiance is the sum of two components:

$$<I(z)> = p_0(z)I_0(z) + p_1(z)I_1(z)$$

p_1 is the percentage cover of the canopy, $p_0 = 1 - p_1$, λ_i are simply the Markov transition lengths depending on the shape of the crown.

NUMERICAL SOLUTIONS

Equations (9) and (10) are the typical form of the integro-differential equations for which an extensive body of numerical solution methods is readily available. In this study the Gauss-Seidel algorithm used in our earlier study (Liang and Strahler, 1993) is applied to solve these two equations. Due to limitation of the space, data analysis will be omitted. Details are available in the separate paper (liang and Strahler, 1994).

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