THE MODIS-N RADIOMETRIC MATH MODEL
Overview

Presented to
MODIS Science Team Calibration Working Group
NASA Goddard Space Flight Center

April 13 - 15, 1992

Attachment 3.12

MODIS-N
GSFC SBRC

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RADIOMETRIC MATH MODEL COMPUTES SENSITIVITY AND ACCURACY

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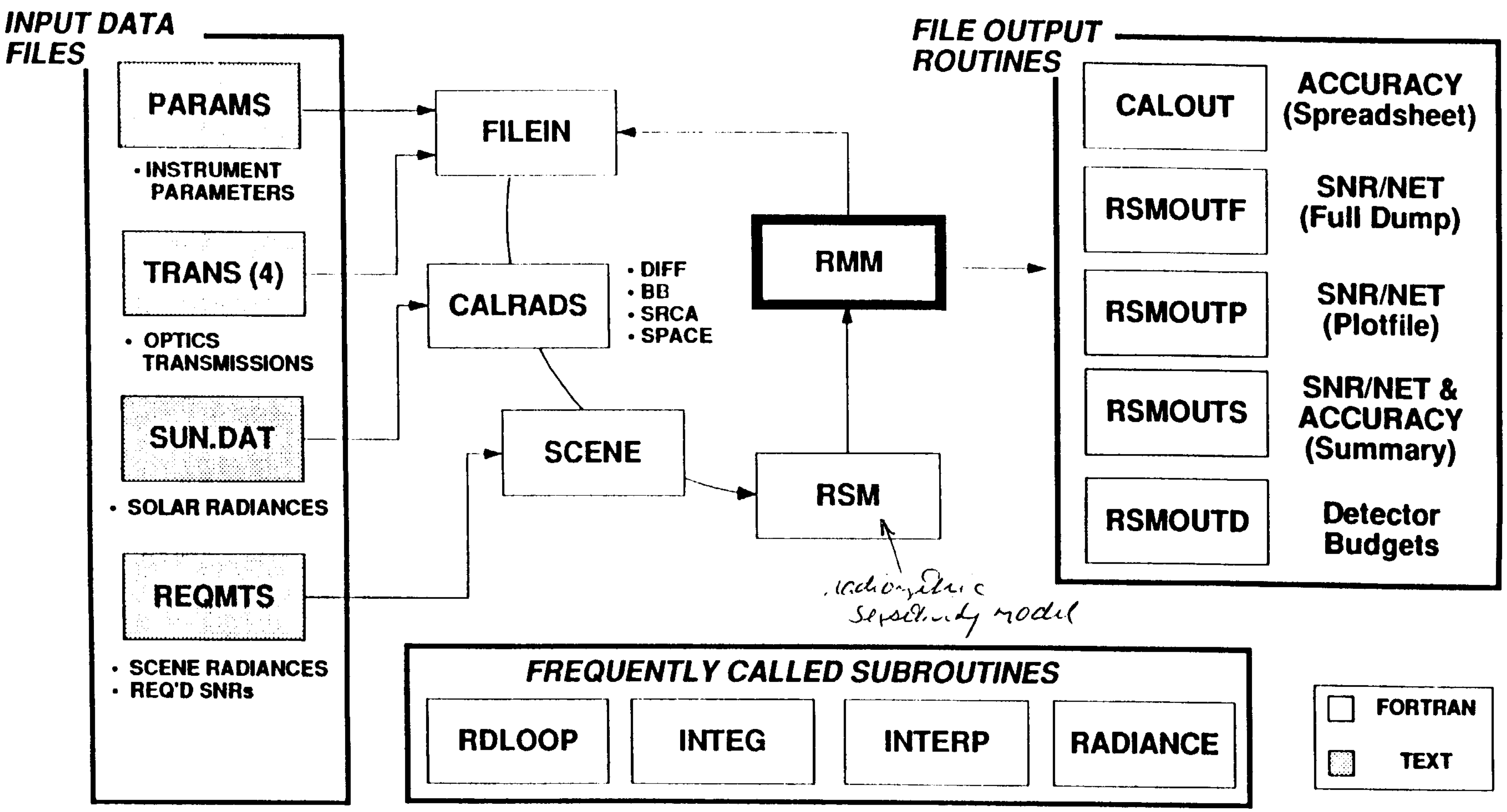

SHORT/MID IR Page 1

MODIS-N
AVERAGE OPTICAL TRANSMISSION
DATA SHEET

5-Mar-92

TITLE:	Short-Wave / Mid-Wave IR
BANDS:	5,6,7,20,21,22,23,24,25,26
Min. Wvlngth.	1.23
Max. Wvlgnth.	4.59

Band	Temp	5	6	7	20	21	22	23	24	25	26
Center Wvlgnth.		**1.24**	**1.64**	**2.13**	**3.75**	**3.75**	**3.959**	**4.05**	**4.465**	**4.515**	**4.565**
Min. Wvlngth.		1.23	1.63	2.105	3.66	3.725	3.934	4.025	4.44	4.49	4.54
Max. Wvlgnth.		1.25	1.65	2.155	3.84	3.775	3.984	4.075	4.49	4.54	4.59
Scan_Mirror	290.0	0.9752	0.9836	0.9871	0.9893	0.9893	0.9896	0.9898	0.9896	0.9896	0.9896
Fold_1	290.0	0.9752	0.9836	0.9871	0.9893	0.9893	0.9896	0.9898	0.9896	0.9896	0.9896
Primary	290.0	0.9717	0.9822	0.9867	0.9896	0.9896	0.9900	0.9901	0.9900	0.9900	0.9900
Secondary	290.0	0.9717	0.9822	0.9867	0.9896	0.9896	0.9900	0.9901	0.9900	0.9900	0.9900
Dichroic_1_(ZnSe)	290.0	0.8124	0.9550	0.9033	0.9770	0.9770	0.9820	0.9697	0.8770	0.8737	0.8714
Dichroic_3_(Mirror)	290.0	0.9890	0.9887	0.9956	0.9983	0.9983	0.9986	0.9987	0.9937	0.9899	0.9846
MW_Lens_1_(ZnSe)	290.0	0.9585	0.9585	0.9585	0.9585	0.9585	0.9585	0.9585	0.9585	0.9585	0.9585
MW_Lens_2_(CdTe)	290.0	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545
Fold_2	290.0	0.9850	0.9850	0.9850	0.9850	0.9850	0.9850	0.9850	0.9850	0.9850	0.9850
MW_Lens_3_(ZnSe)	290.0	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595
Roof_Mirror	290.0	0.9702	0.9702	0.9702	0.9702	0.9702	0.9702	0.9702	0.9702	0.9702	0.9702
MW_Lens_4_(ZnSe)	290.0	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595	0.9595
MW_Lens_5_(ZnSe)	290.0	0.9592	0.9592	0.9592	0.9592	0.9592	0.9592	0.9592	0.9592	0.9592	0.9592
MW_Window_1_(Saph)	290.0	0.9500	0.9500	0.9500	0.9500	0.9500	0.9500	0.9500	0.8930	0.8930	0.8930
MW_Window_2_(Saph)	140.0	0.8900	0.8900	0.8900	0.8900	0.8900	0.8900	0.8900	0.8366	0.8366	0.8366
MW_Window_3_(Saph)	85.0	0.9500	0.9500	0.9500	0.9500	0.9500	0.9500	0.9500	0.8930	0.8930	0.8930
Band_Pass_Filter	85.0	0.7000	0.7000	0.8000	0.8000	0.8000	0.8000	0.8000	0.8000	0.8000	0.8000
Transmission		**0.3132**	**0.3825**	**0.4233**	**0.4638**	**0.4638**	**0.4669**	**0.4614**	**0.3447**	**0.3421**	**0.3394**

Notes:

1) Silver mirror reflectance data is a design estimate by S. Pellicori

2) Transmission coefficient of dichroic 1 and 3 are design estimates from S. Pellicori.

3) Band pass filter performance are from SBRC Specification # E85146

4) Assumed window material is Saphire

5) SWIR/MWIR lens ARC assumed performance (from S. Pellicori): 98% per surface

6) Aft optics mirrors are assumed to be gold coated (R=98.5%) with 2 reflections in the roof mirro

7) Intermediate window performance is a preliminary estimates from S. Pellicori.

8) Bands 24-26 have 4% internal absorption for saphire.

Created on 3/4/92

3/5/92 7:3 M

SOLID ANGLE MODEL USED TO COMPUTE BACKGROUND CAN BE INCORPORATED INTO RMM

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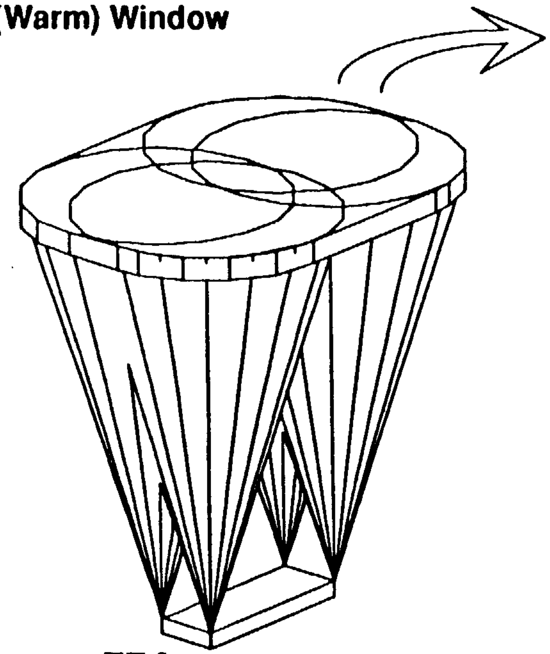
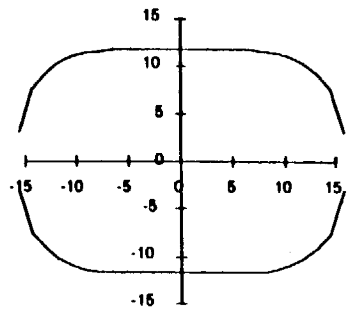
Radiative Cooler
1st Stage
(Warm) Window

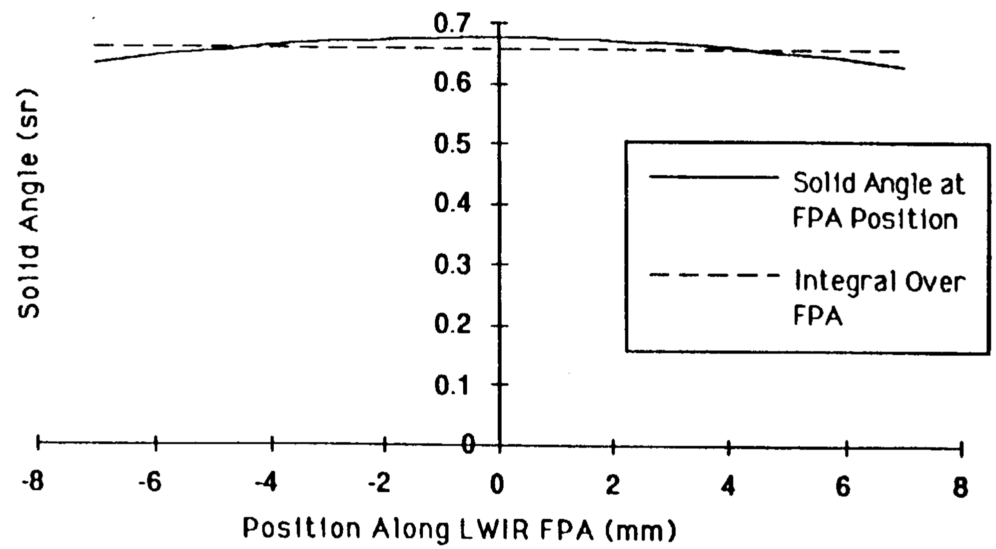




FPA

F-CONE Defines Window Geometry

- MODEL INTEGRATES OVER TARGET AND RECEIVER SURFACES
- COMPUTES AΩ PRODUCT
- CAN BE USED TO COMPUTE Ω's OF SCATTERED/STRAY LIGHT ON CAL TARGETS IN RMM



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RADIOMETRIC SENSITIVITY

SNR, NEΔT

MODIS-N

GSFC SBRC

MARGINS EXCEED SPECS IN ALL BANDS

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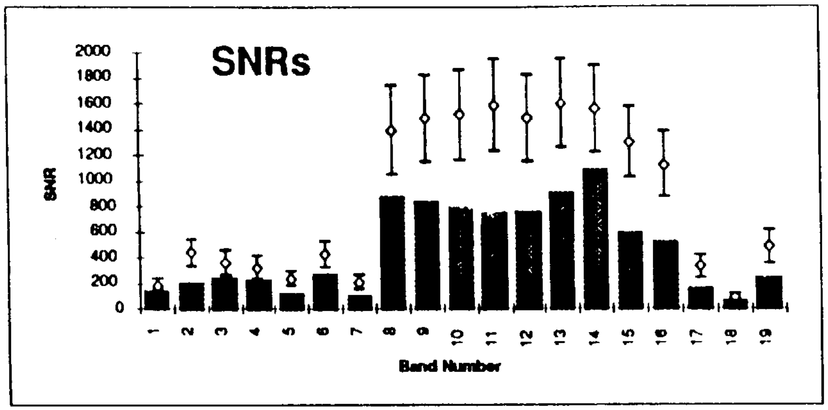
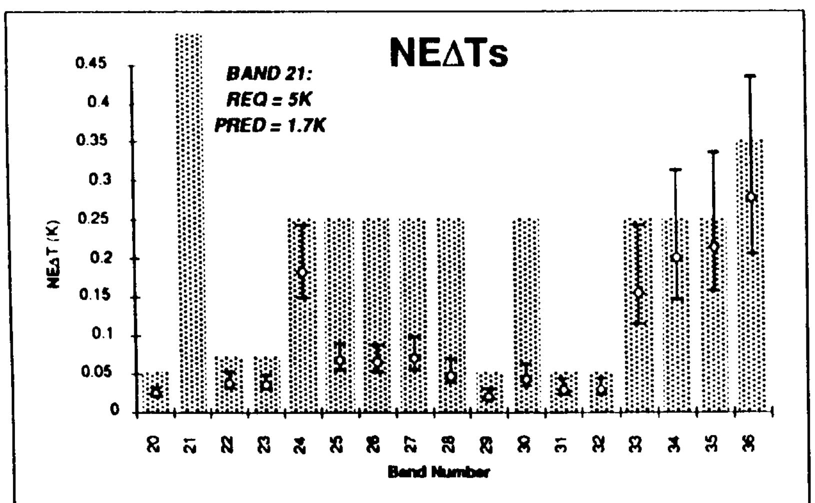
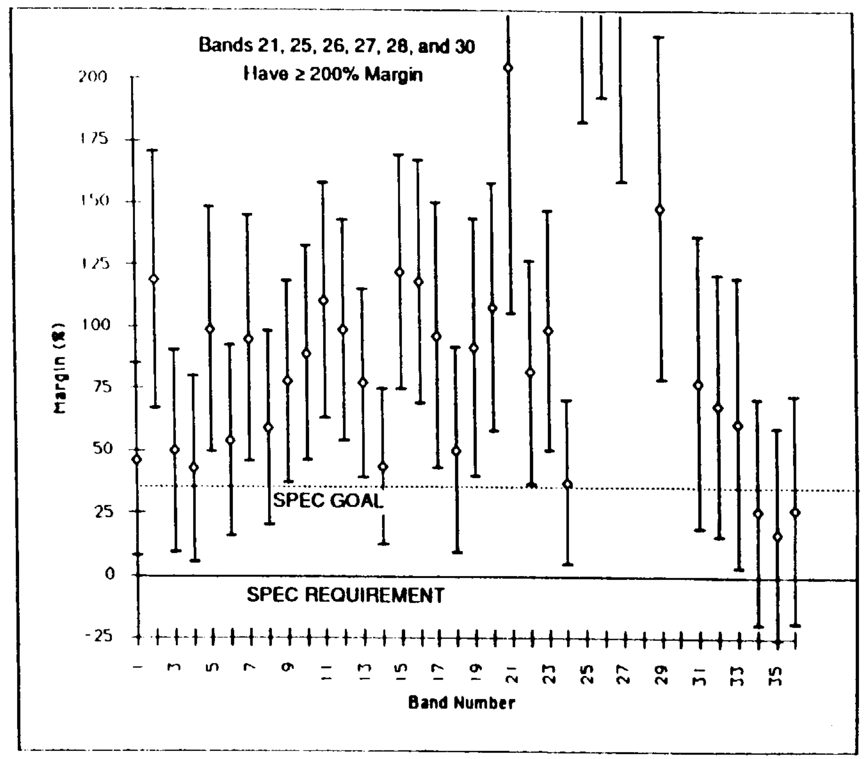




MARGINS



- ERROR BARS REPRESENT 3 SIGMA UNCERTAINTY

REFLECTIVE BANDS COVER WIDE SIGNAL RANGE

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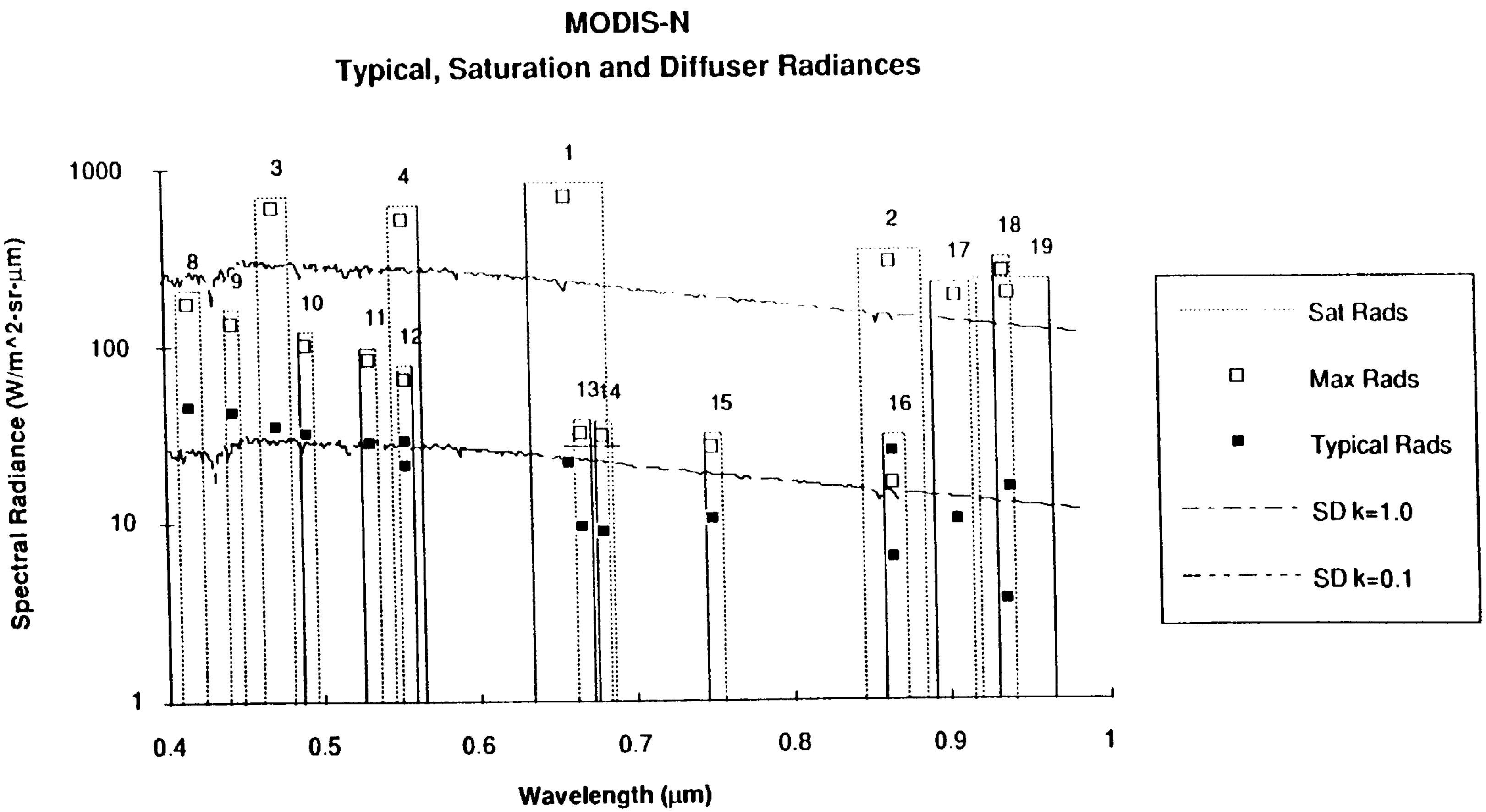

SIGNAL LEVELS FOR SWIR BANDS

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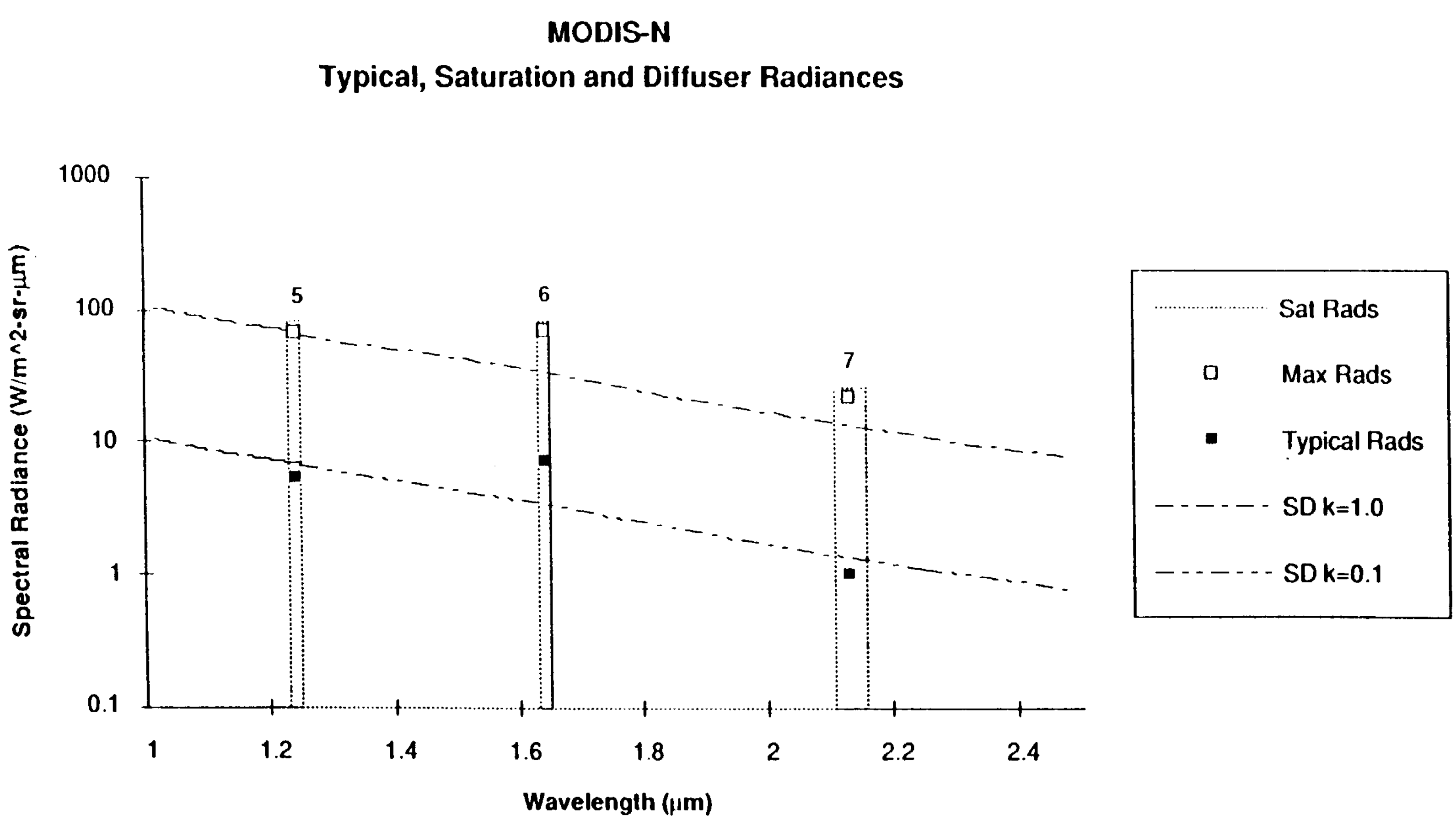

EMISSIVE BANDS ACCOMMODATE FULL RANGE OF SIGNAL LEVELS

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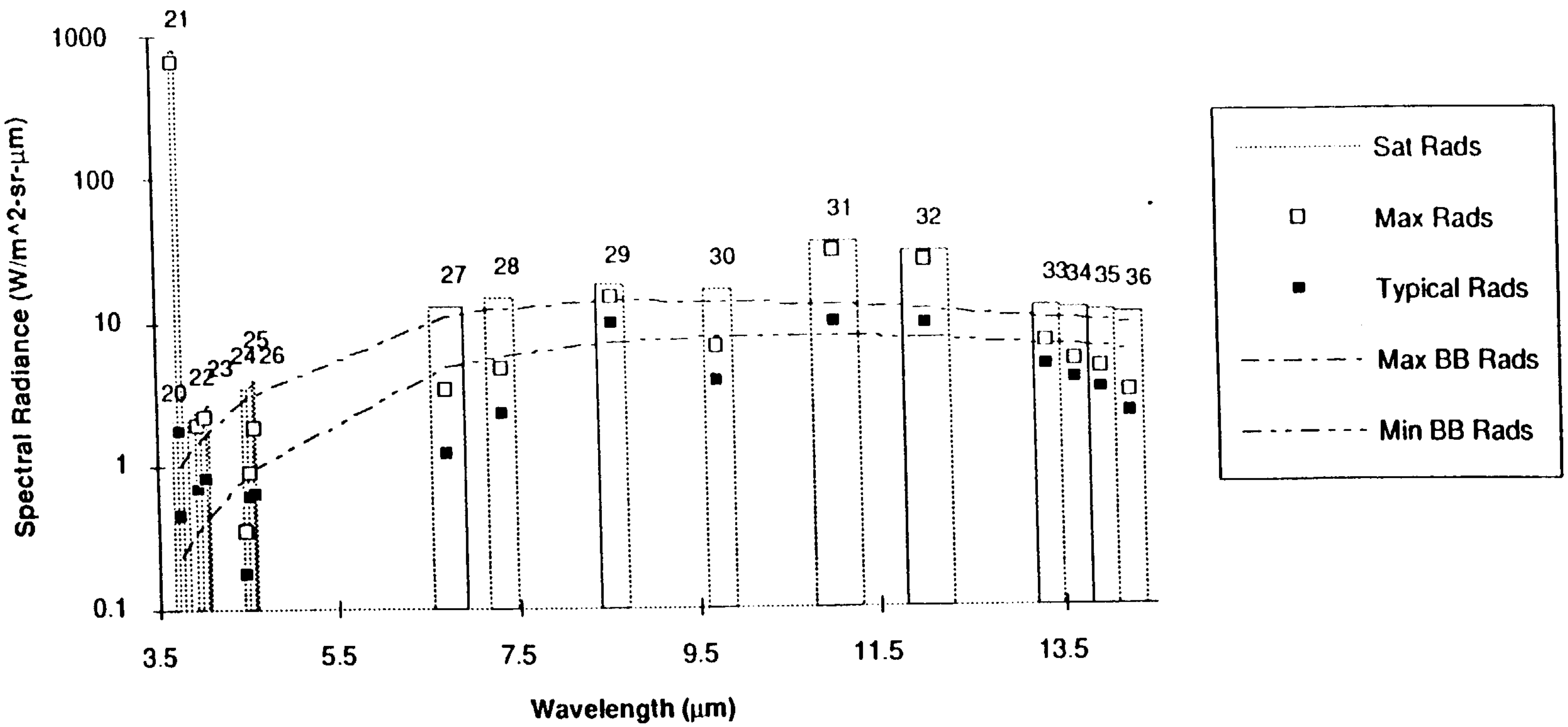
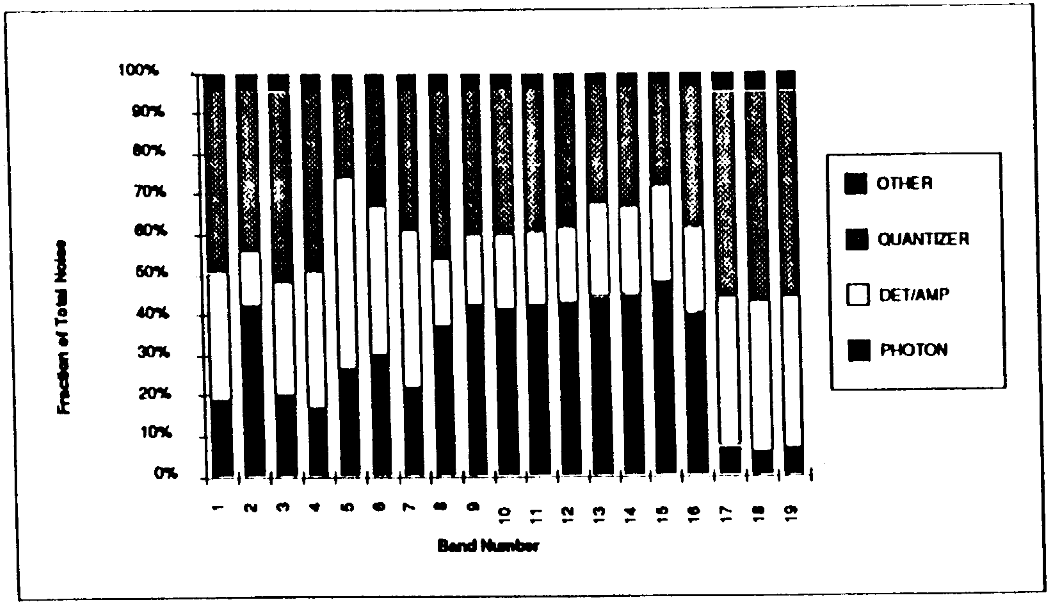
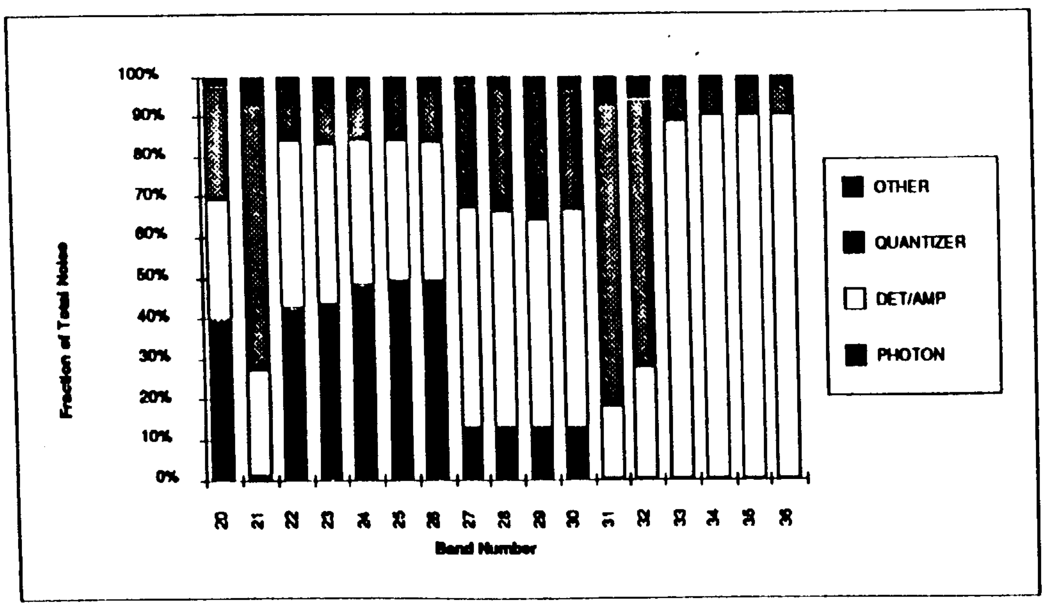
Typical, Saturation and Blackbody Radiances







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NOISE LEVELS FOR MODIS-N REFLECT SIGNAL AND DYNAMIC RANGE REQ'D

- QUANTIZATION AFFECTS 21, 31, 32
- HIGH DETECTOR NOISE IN 33-36

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RADIOMETRIC ACCURACY

REFLECTIVE, EMISSIVE BANDS

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RADIOMETRIC MATH MODEL INCLUDES MANY CONTRIBUTORS

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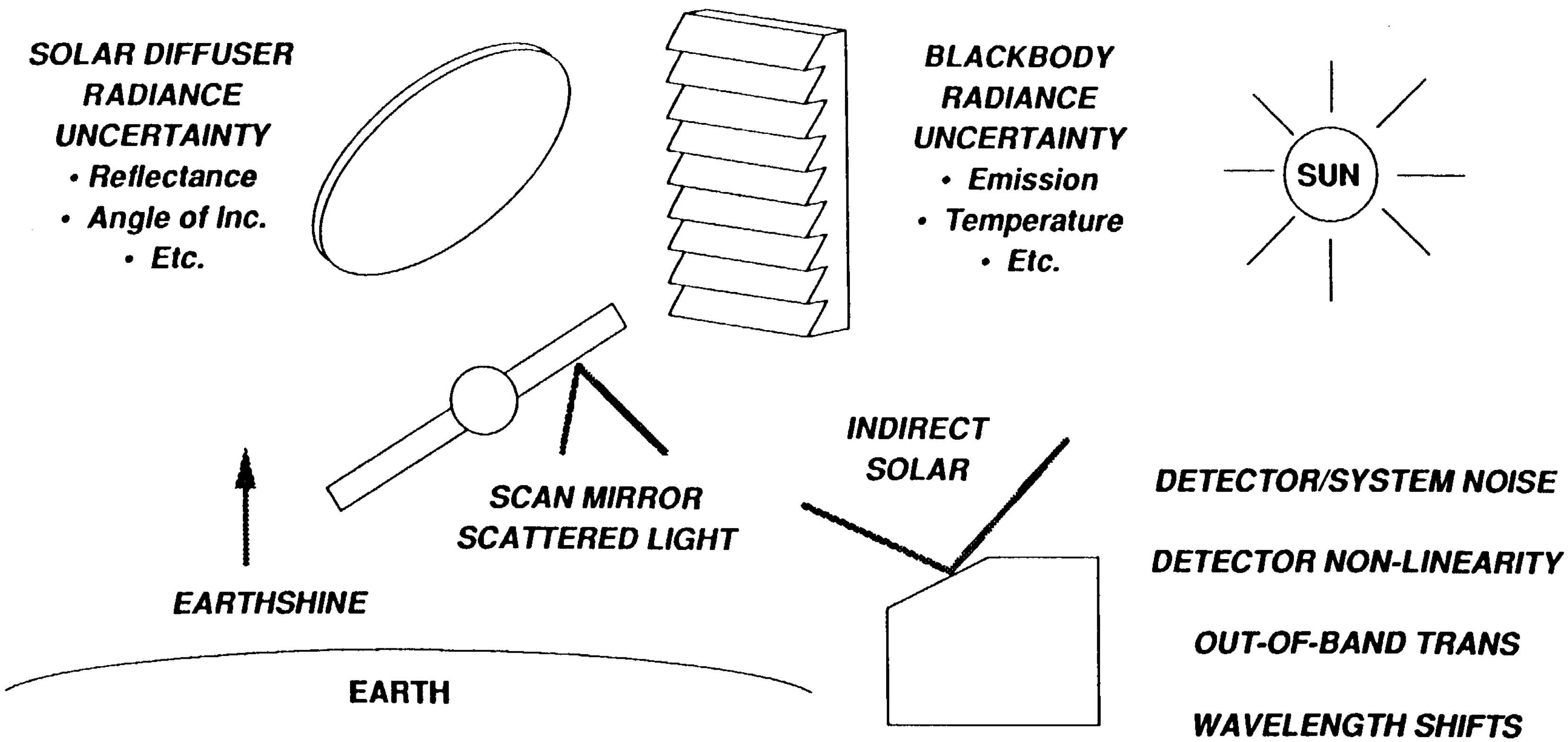

SNR

NEΔT

RADIOMETRIC ACCURACY

MODIS-N GSFC SBRC

REFLECTIVE BAND IN-FLIGHT RADIOMETRIC ACCURACY ASSUMPTIONS

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SCENE

- SINGLE PIXEL BASIS (NO AVERAGING OF SCENE DATA)
- SCENE UNIFORM ACROSS SAMPLE (NO MTF ERRORS)
- NO SPECTRAL BAND REGISTRATION ERRORS
- 50% SCENE POLARIZATION, 2% INSTRUMENT

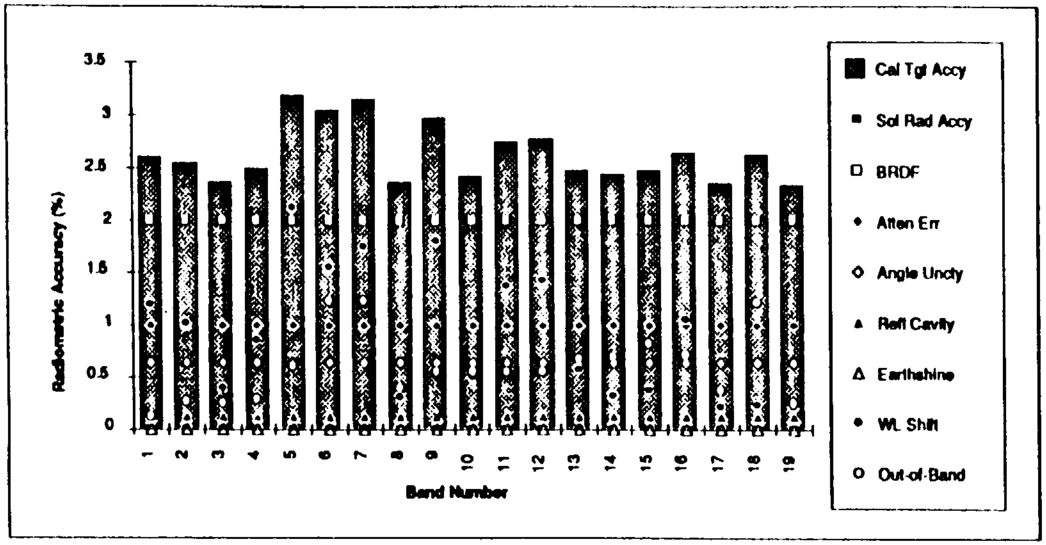
DIFFUSER

- NO SOLAR IRRADIANCE UNCERTAINTY
- BRDF: $1/\pi \pm 2\%$
- AOI SUN ON DIFFUSER: $62.6° \pm 0.3\%$
- 15 SAMPLES AVERAGED ON SOLAR DIFFUSER
- SCREEN TRANSMISSION: $0.1 \pm 1\%$
- NO INDIRECT SOLAR
- NO EARTHSHINE ON SOLAR DIFFUSER

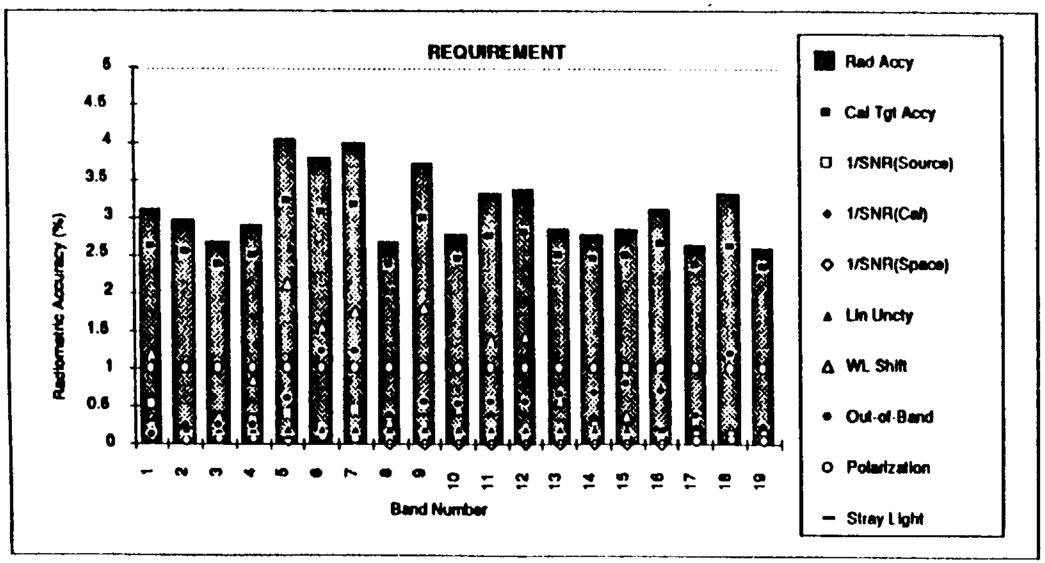
INSTRUMENT

- UNCORRELATED WAVELENGTH SHIFT: SCENE TO DIFFUSER
- OUT OF BAND TRANSMISSION: 0.0001
- BACKSCATTERED ENERGY: 0.13%
- 0.2% KNOWLEDGE OF TRANSFER FUNCTION (LINEARITY)
- 0.2% SCAN MIRROR SCATTER, $\Omega = 2$ sr, rhoEarth = 50%

ACCURACY OF DIFFUSER RADIANCE



ACCURACY OF SCENE RADIANCE



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REFLECTIVE IN-FLIGHT RADIOMETRIC ACCURACY MEETS SPECS IN ALL BANDS

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EMISSIVE BAND IN-FLIGHT RADIOMETRIC ACCURACY ASSUMPTIONS

SCENE

- SINGLE PIXEL BASIS (NO AVERAGING OF SCENE DATA)
- SCENE UNIFORM ACROSS SAMPLE (NO MTF ERRORS)
- NO SPECTRAL BAND REGISTRATION ERRORS
- NO POLARIZATION ERRORS

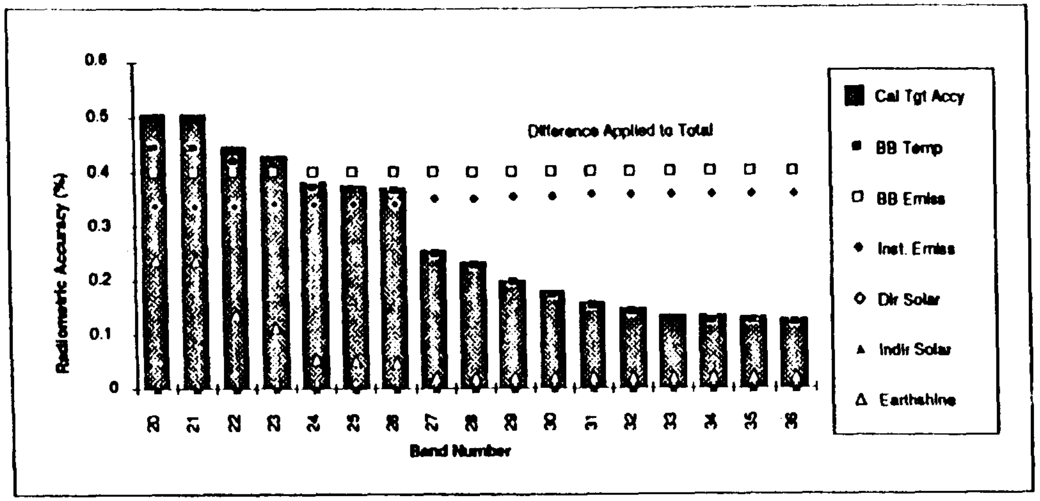
BLACKBODY

- EMISSIVITY: 0.992 ± 0.4%
- BLACKBODY TEMPERATURE: 295K ± 0.1K
- 15 SAMPLES AVERAGED ON BLACKBODY
- NO DIRECT SOLAR ON BLACKBODY
- NO INDIRECT SOLAR ON BLACKBODY
- EARTHSHINE ON BLACKBODY: Ω = 0.081 sr, T=295K Rho = 50%

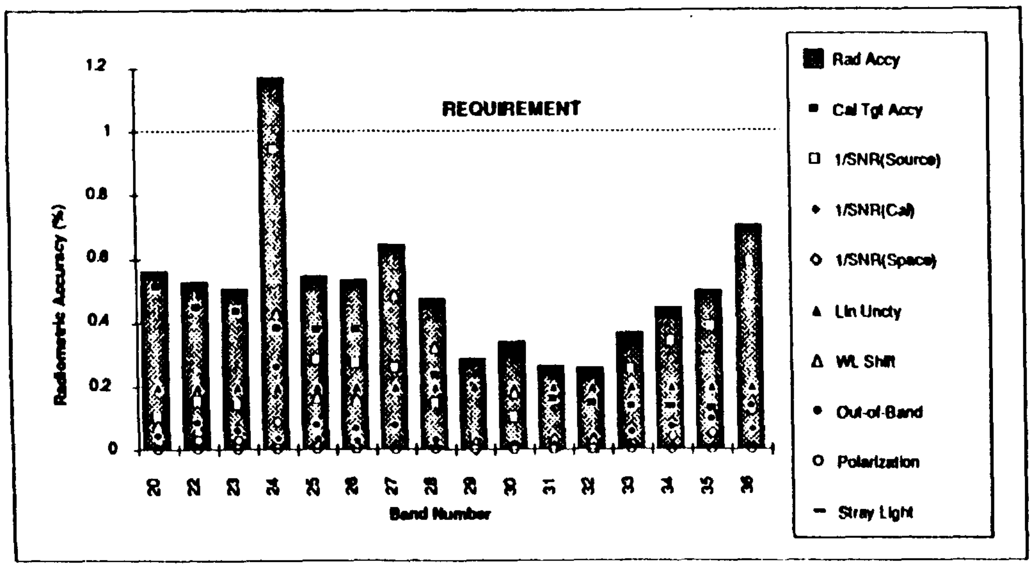
INSTRUMENT

- INSTRUMENT TEMPERATURE: 293K
- OUT OF BAND TRANSMISSION: 0.001
- CORRELATED WAVELENGTH SHIFT OF SCENE AND BLACKBODY
- CORRELATED OUT-OF-BAND OF SCENE AND BLACKBODY
- CORRELATED INSTRUMENT AND BLACKBODY EMISSIONS
- 0.2% KNOWLEDGE OF TRANSFER FUNCTION (LINEARITY)
- 0.2% SCAN MIRROR TOTAL INTEGRATED SCATTER

ACCURACY OF BLACKBODY RADIANCE



ACCURACY OF SCENE RADIANCE



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EMISSIVE IN-FLIGHT RADIOMETRIC ACCURACY MEET WITH MARGIN IN MOST BANDS

- BAND 24 LIMITED BY SNR AT Ltyp

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FUTURE REFINEMENTS

SUMMARY AND CONCLUSIONS

FUTURE REFINEMENTS FOR THE RADIOMETRIC MATH MODEL

- USER-FRIENDLY INTERFACE
  - MENUS
  - GRAPHICS
  - ALLOW LOOPS FOR ITERATIONS
- INCORPORATE SRCA RADIANCE MODEL TO "CALRADS"
- INCORPORATE SOLID ANGLE MODEL
- INCLUDE BANDPASS FILTER PROFILES FOR ALL BANDS
- POLARIZATION
  - USE CURRENT PREDICTIONS FOR ALL BANDS
  - INCORPORATE PHASE CAPABILITY
- SCAN MIRROR AFFECTS: REFLECTANCE WITH SCAN ANGLE
- 1/f NOISE FOR PC BANDS TO BE IN TERMS OF NOISE AT 1 Hz (CURRENTLY IN TERMS OF Fknee)

SUMMARY AND CONCLUSIONS

- RADIOMETRIC MATH MODEL "ENGINE" RUNNING
- INCLUDES CALCULATION OF SENSITIVITY AND ACCURACY
- MANY CONTRIBUTORS ACCOUNTED FOR
- PRELIMINARY BUDGETS SHOW ACCURACY SPEC DIFFICULT FOR SOME BANDS
- ADDITIONAL INFO NEEDED FOR CAL INPUT VARIABLES
- MODEL REFINEMENT IN PROGRESS CONTINUALLY

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INTERNAL MEMORANDUM

TO: J. Young

CC: G. Barnett, L. Candell, R. Durham, J. Engel

DATE: April 6, 1992
REF: PL3095-N00910
FROM: T.S. Pagano
BLDG: B32
MAIL STA: 79
EXT: 7343

SUBJECT: **Refined Accuracy Analysis Model and Results**

Introduction

This memorandum supplements technical memoranda PL3095-M00260, PL3095-N00620 and PL3095-N00736. It represents the current theory used in the radiometric accuracy modeling of the RMM. Most of the analysis was presented in the RMM technical review package of March 11, 1992 (see PL3095-00736). It is presented again here in its entirety with three major modifications.

1. Correlated out-of-band radiance of the scene and the blackbody. (Previously Uncorrelated)

2. Correlated Instrument Emission and Blackbody Emission in the Infrared. (Previously Uncorrelated)

3. Addition of Backscattered Solar Diffuser Energy

Radiometric Accuracy Performance Modeling

The radiance equation relates the radiance of the calibrator to the signals obtained when viewing the scene and the calibrators and the scattered light entering the aperture.

$S_{SC} - S_{SP}$

$$L_{SC} = \left(\frac{\Delta S_{SC}}{\Delta S_{CAL}}\right) \cdot L_{CAL} - L_{SCAT}$$

radiance of scene $\quad$ $S_{cal} - S_{SP}$

Earlier we showed (PL3095-N00478) that the relative uncertainty in the scene radiance can be related to the relative uncertainty of the calibration target and the signal-to-noise ratios of the system. This was found by using the classical error propagation formula and differentiating with respect to the scene signal, the blackbody signal, the space view signal, and the calibrator radiance. Recently terms have been included to account for wavelength shifts, polarization and scattered light off the scan mirror. The resulting expression, **the radiometric accuracy equation**, is given below. Note that all errors except the wavelength shift and out-of-band response for the emissive bands are assumed to be uncorrelated.

$$\left(\frac{\partial L_{SC}}{L_{SC}}\right)^2 = \left(\frac{1}{SNR_{SC}}\right)^2 + \left(\frac{\partial S_0}{\Delta S_{CAL}} - \frac{\partial S_0}{\Delta S_{SC}}\right)^2 + \left(\frac{1}{SNR_{CAL}}\right)^2 + \left(\frac{\partial L_{CAL}}{L_{CAL}}\right)^2 + (\varepsilon_{SC})^2$$

variance of signal

blackbody radiance uncertainty

$$+\left[\left(\frac{\partial L_{SC}}{\partial\lambda} - f(I)\frac{\partial L(T_{BB},\lambda)}{\partial\lambda} \cdot \frac{L_{SC}}{L(T_{BB},\lambda)}\right)\frac{\Delta\lambda}{L_{SC}}\right]^2 + P\left(\frac{L_{SCs} - L_{SCp}}{L_{SC}}\right)^2 + \left(\frac{L_{SCAT}}{L_{SC}}\right)^2$$

$$+\frac{T_{oob}}{L_{SC}}\left\{\left[L(T_{SC},\lambda)_{tot} - L(T_{SC},\lambda)\right] - f(I)\left[L(T_{BB},\lambda)_{tot} - L(T_{BB},\lambda)\right] \cdot \frac{L_{SC}}{L(T_{BB},\lambda)}\right\}^2$$

where,

L_{SC} = Scene Radiance (s and p denote polarization states) [W/m^2 -sr - µm]
L_{CAL} = Calibrator Radiance [W/m^2 -sr - µm]
SNR_{SC} = Scene Signal divided by All Noise (including 1/f of the data at the end of scan)
SNR_{SP} = Scene Signal divided by Noise in the Space Port
SNR_{CAL} = Calibrator Signal divided by All Noise when viewing Blackbody
∂S_o = System Noise in the Space Port
$\Delta S_{CAL} = S_{CAL} - S_o$ = Calibrator Signal With the Space Signal Offset Subtracted
$\Delta S_{SC} = S_{SC} - S_o$ = Scene Signal With the Space Signal Offset Subtracted
ε_{SC} = Uncertainty in the System Response Curve (Transfer Function) at the Scene Radiance
λ = Center Wavelength of the Bandpass [µm]
$\Delta\lambda$ = Shift of the Center Wavelength of the Bandpass [µm]
I = Band Number
f(I) = 0 for Reflective Bands ($I \leq 19$), 1 for Emissive Bands ($I \geq 20$)
T_{BB} = Calibration Blackbody Temperature
P = Polarization Factor of the Instrument
L_{SCAT} = Scattered Light (Primarily from the Scan Mirror) [W/m^2 -sr - µm]
T_{oob} = Out-of-Band Transmission
tot = Represents Radiance Integrated over Total Range of Detector Response

The wavelength shift term shows a direct correlation between the radiances of the scene and the blackbody calibrator. It assumes that if a wavelength shift occurs, the radiance of the calibrator and the scene will shift together. This correlation is not used in the case of the reflective bands for the solar diffuser. In the case of the reflective bands, the term f(I) = 0, and the wavelength shift errors of the solar diffuser radiances are treated separately (see below). The wavelength shift on the calibrator is normalized to the scene radiance.

The second to last term says that for a completely polarized scene radiance, and no other errors, the error in the scene radiance will be the polarization factor of the instrument, P.

The radiometric accuracy of a given band is then found by computing the SNR on the calibrator, the SNR of the scene, the noise in the space view port, the uncertainty of the radiance coming from the calibration target, the nonlinearity, the potential wavelength shift, the polarization and the scattered light.

Scattered Light off the Scan Mirror

The scan mirror will scatter radiation from the Earth when the instrument is viewing the scene. The Earth scene radiation is calculated for the reflective bands using an average Earth albedo and the solar irradiances, assuming a Lambertian BRDF ($1/\pi$ sr). The scattered light off the scan mirror is then

$$L_{SCAT} = L_{Earth}\,\Omega_{E/SM}\,\frac{TIS}{\pi}$$

where

L_{Earth} = Spectral Radiance at the Scan Mirror From the Earth [W/m^2-sr-μm]
$\Omega_{E/SM}$ = Solid Angle of the Earth as Viewed by the Scan Mirror [sr]
TIS = Total Integrated Scatter ≈ 0.002 to 0.0005

The low total integrated scatter for the scan mirrors (taken from Thematic Mapper Data) makes this term relatively small as an error contributor.

Nonlinearity at Scene Radiance

The term ε_{SC} represents the uncertainty in the system response curve at the scene radiance. The magnitude of this term depends on a number of factors. First, how well can we calibrate the response curve in the laboratory before flight. Secondly, once we have the calibration curve, how well can we extrapolate back to the scene radiance, from the calibrator radiance. A high degree of nonlinearity in the system transfer function will increase our uncertainty in the system response curve.

Scene Radiance Gradient w.r.t. Wavelength and Temperature

A wavelength uncertainty will cause an error proportional to the magnitude of the wavelength shift and the slope of the scene radiance with respect to wavelength. The derivative of the scene radiance can be obtained from the solar spectrum in the reflective bands, and the blackbody function in the emissive bands. For the reflective bands,

$$\frac{\partial L_{SC}}{\partial \lambda}\,\mathrm{Refl} = \frac{E_{sun}(\lambda_2) - E_{sun}(\lambda_1)}{(\lambda_2 - \lambda_1)}$$

where

$$\lambda_1 = \lambda_{Cent} - \Delta\lambda/2 \qquad \text{and} \qquad \lambda_2 = \lambda_{Cent} + \Delta\lambda/2$$

For the emissive bands the scene radiance is related to the temperature by the blackbody equation

$$L(T,\lambda) = \frac{2\pi hc^2}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda kt} - 1}\cdot\frac{1}{\pi} \qquad [W/m^2\text{-sr-}\mu m]$$

Define

$c_1 = 2\pi hc^2$, and $c_2 = hc/k$, then differentiate with respect to wavelength, we get,

$$\frac{dL(T,\lambda)}{d\lambda} = \frac{1}{\pi}\left(\frac{c_1 c_2}{\lambda^7 T}\,\frac{e^{c_2/\lambda t}}{(e^{c_2/\lambda t}-1)^2} - \frac{5\,c_1}{\lambda^6}\,\frac{1}{e^{c_2/\lambda t}-1}\right)$$

Another useful equation is the temperature derivative of the plank blackbody equation,

$$\frac{dL(T,\lambda)}{dT} = \frac{1}{\pi}\,\frac{c_1 c_2}{\lambda^6 T^2}\,\frac{e^{c_2/\lambda t}}{(e^{c_2/\lambda t}-1)^2}$$

When we talk about integrating the blackbody equation, over a wavelength interval, we actually take the integrated average of the corresponding equation above. For example, to integrate the temperature derivative equation over the bandpass,

$$\frac{dL(T,\lambda_{Cent})}{dT} = \frac{1}{\Delta\lambda}\int_{\lambda_1}^{\lambda_2}\frac{dL(T,\lambda)}{dT}\,d\lambda$$

Solar Diffuser Radiance Uncertainty

The terms in the radiometric accuracy equation include uncertainties present from the main instrument, except the radiometric uncertainty in the in-flight calibrator radiance, $\frac{\partial L_{CAL}}{L_{CAL}}$. This uncertainty depends on the uncertainty in the terms that make up the radiance of the calibrator. For the case of the in-flight solar diffuser, the radiance is given by

$$L_{SD} = E_{Sun}\,\beta\,\cos\theta\;K - \Delta L_{SD}$$

where

L_{SD} = Radiance of the Solar Diffuser at the MODIS-N Aperture in the Wavelength of the Band [W/m^2 -sr - µm]
E_{Sun} = Irradiance of the Sun at the Solar Diffuser at the Wavelength of the Band [W/m^2 - µm]
β = BRDF of the Solar Diffuser (assumed angle independent) = $1/\pi$ [sr^{-1}]
θ = Angle Between the Sun and the Normal to the Solar Diffuser [r]
K = Attenuation Constant for the Illuminated Diffuser
ΔL_{SD} = Scattered/Stray/Spurious Radiance from the Diffuser [W/m^2 -sr - µm]

Applying the variance analysis on the solar diffuser radiance equation with respect to all of the parameters and wavelength, assuming E_{Sun} has the only wavelength dependence, we get for the relative solar diffuser radiance uncertainty (assumes ΔL_{SD} and out-of-band radiance contributions « L_{SD})

$$\left(\frac{\partial L_{SD}}{L_{SD}}\right)^2 = \left(\frac{\partial E_{Sun}}{E_{Sun}}\right)^2 + \left(\frac{\partial \beta}{\beta}\right)^2 + \left(\frac{\partial K}{K}\right)^2 + \left(\tan\theta\ \partial\theta\right)^2$$

$$+\left(\frac{\Delta L_{SD}}{E_{Sun}\ \beta\ \cos\theta\ K}\right)^2 + \left(\frac{\partial E_{Sun}}{\partial\lambda}\ \frac{\Delta\lambda}{E_{Sun}}\right)^2 + \left(T_{oob}\frac{E_{Sun\text{-}tot} - E_{Sun}}{E_{Sun}}\right)^2$$

Notice that the scattered light term, ΔL_{SD}, has been treated as a total error, i.e. we do not know what it will be at any moment when viewing the scene. The last term includes the integrated out-of-band transmission, T_{oob}, multiplied by the relative magnitude of the out-of-band solar irradiance to the in-band irradiance.

This equation for $\frac{\partial L_{SD}}{L_{SD}}$ is then to be substituted into the radiometric accuracy equation for $\frac{\partial L_{CAL}}{L_{CAL}}$ for bands 1 through 19 (reflective bands).

Scattered/Stray/Spurious Radiance from the Diffuser

All of the terms in the above equation are straightforward, except for the scattered light term, ΔL_{SD}. We need to ask ourselves what are the sources that could possibly contribute radiant energy falling on the diffuser. Three possible contributors are indirect solar, backscattered solar and earthshine. Indirect solar results when solar strikes a part of the instrument that is in the field of view of the solar diffuser. Backscattered solar strikes the diffuser, is reflected off, strikes the instrument, then comes back for a second reflection off the diffuser. Earthshine is the Earth scene radiance that is derived from the portion of the Earth that is seen by the diffuser.

$$\Delta L_{SD} = E_{Sun}\ \beta\ K\left(\cos\theta_{SC/SD}\ \rho_{MOD}\ \frac{\Omega_{Scat}}{\pi} + \cos\theta\ \pi\ \beta\ \rho_{MOD}\ F_{SD}\ F_W\right) + L_{Earth}\ \Omega_{E/SD}\ \beta\ \cos\theta_{E/SD}$$

where the new terms are

$\theta_{SC/SD}$ = Average Angle Between the Solar Diffuser and the Incident Indirect Solar [r]
ρ_{MOD}= In-Band Reflectance of MODIS Surface off Which Indirect Solar is Incident
Ω_{Scat} = Solid Angle of the Scattered Radiant Energy As Seen from the Solar Diffuser [sr]
L_{Earth} = Average Radiance of the Earth-shine on the Diffuser [W/m^2 -sr - μm]
$\Omega_{E/SD}$ = Solid Angle of the Earth-shine As Seen from the Solar Diffuser [sr]
$\theta_{E/SD}$ = Average Angle Between the Solar Diffuser and the Incident Earth-shine [r]
F_{SD} = Fraction of Solar Diffuser Area to Total Scan Cavity Area
F_W = Fraction of the Cavity Area that Can Potentially Reflect Diffuser Radiation

It is assumed that the out-of-band contributors to the signal from these sources is negligible.

Blackbody Radiance Uncertainty

The blackbody radiance uncertainty can be found in much the same way as done for the solar diffuser. The radiance of the blackbody is given by

$$L_{BB} = \varepsilon\ L(T_{BB},\lambda) - \Delta L_{BB}$$

where

L_{BB} = Radiance of the Blackbody at the MODIS-N Aperture in the Wavelength of the Band [W/m^2 -sr - µm]
ε = Emissivity of the Blackbody
$L(T_{BB},\lambda)$ = The Plank Blackbody Distribution Function Integrated Over the Band [W/m^2 -sr - µm]
ΔL_{BB} = Scattered/Stray/Spurious Radiance from the Blackbody [W/m^2 -sr - µm]

When we apply the variance analysis to the blackbody radiance, we differentiate with respect to the emissivity of the blackbody, and the temperature. Here we have correlated the emission from the blackbody and the instrument in the first term on the RHS of the equation. This formula implies that if the blackbody and the instrument are at the same temperature, and the solid angle of the instrument as seen by the blackbody is near π, the emissivity error will be small. This is like treating the scan cavity as a blackbody integrating cavity. The wavelength shift and out-of-band response is included in the radiometric accuracy equation above and is therefore not included again here. The result for the relative blackbody radiance uncertainty is (for $\Delta L_{BB} \ll L_{BB}$)

$$\left(\frac{\partial L_{BB}}{L_{BB}}\right)^2 = \left[\frac{\partial\varepsilon}{\varepsilon} \bullet \left(1 - \frac{L(T_{Inst},\lambda)}{L(T_{BB},\lambda)}\frac{\Omega_{Inst}}{\pi}\right)\right]^2 + \left(\frac{\partial L(T_{BB},\lambda)}{\partial T}\frac{\Delta T}{L(T_{BB},\lambda)}\right)^2 + \left(\frac{\Delta L_{BB}}{\varepsilon\, L(T_{BB},\lambda)}\right)^2$$

Scattered/Stray/Spurious Radiance from the Blackbody

The undesirable energy from the blackbody can be categorized into four major areas:

- Reflection of direct solar energy onto the blackbody
- Indirect solar energy that strikes the blackbody after reflecting off the internal MODIS-N cavity
- Emission from the MODIS-N internal cavity itself
- Reflection of direct Earth-shine onto the blackbody

We can sum these sources to get the total undesired energy falling on the blackbody.

$$\Delta L_{BB} = (1-\varepsilon)\, L(T_{Sun},\lambda)\,\frac{\Omega_{Sun}}{\pi}\, F_{ill}\cos\theta_{id} + (1-\varepsilon)\, L(T_{Sun},\lambda)\,\rho_{MOD}\frac{\Omega_{Sun}\,\Omega_{S/R}}{\pi^2} +$$

$$(1-\varepsilon)\, L(T_M,\lambda)\,\varepsilon_{MOD}\frac{\Omega_M}{\pi} + (1-\varepsilon)\, L_{Earth}\frac{\Omega_{E/BB}}{\pi}$$

where the new terms are

$L(T_{Sun},\lambda)$ = Radiance of a blackbody (the sun) of T_{Sun}= 5900K [W/m^2 -sr - µm]
Ω_{Sun} = Solid Angle of the Sun as Seen from the Earth (MODIS-N orbit) = 6.76×10^{-5} [sr]
F_{ill} = Fraction of Blackbody Illuminated by Direct Solar
θ_{id} = Angle of Incidence of Direct Solar on Blackbody [r]
ρ_{MOD}= Reflectance of the Surface Off Which the Solar Energy is Reflecting Before Striking the BB
$\Omega_{S/R}$ = Solid Angle of the Surface Off Which the Solar Energy is Reflecting As Seen by the BB [sr]

T_M = Mean Temperature of the Surfaces of the MODIS-N Emitting onto the BB [K]
ε_{MOD} = Emissivity of the MODIS-N Instrument
Ω_M = Solid Angle of the Surfaces of the MODIS-N Emitting onto the BB As Seen by the BB [sr]
L_{Earth} = Earth Radiance (Reflected Solar and Emitted Blackbody) [W/m^2 -sr - µm]
$\Omega_{E/BB}$ = Solid Angle of the Earth in the FOV of the BB [sr]

In each case, the "reflectance" of the blackbody (1- ε) is used since the sources of the radiance are not from the blackbody itself.

The Earth-shine radiance the sum of the Earth emission and solar reflection:

$$L_{Earth} = L(T_{Earth},\lambda) + L(T_{Sun},\lambda)\,\frac{\Omega_{Sun}}{\pi}\,\rho_{Earth}$$

where

T_{Earth} = Average Temperature of Earth in FOV Observed by BB [K]
ρ_{Earth} = Average Reflectance of Earth in FOV Observed by BB

Summary and Conclusions

The radiometric math model calculates Radiometric Sensitivity and Accuracy for the MODIS-N in its current form. Radiometric Sensitivity or Signal-to-Noise Ratio (SNR) is calculated using techniques documented in earlier memoranda. System level radiometric accuracy depends on the system SNR, and how well we can characterize the signal transfer function. It depends on the uncertainty in the calibration target radiance, and spectral, polarization and scattering properties of the instrument. The Radiometric Accuracy Equation incorporates the dominant uncertainty contributors to the scene radiance.

The Radiometric Math Model, in its current form, it is useful for determining the effects of various design configurations on the overall radiometric performance. Later it will evolve to function as a MODIS-N simulator, combining actual test results of system and subsystem parameters, and allowing determination of system phenomenon from the observables. Input into the types of capabilities desired of the RMM should be made as soon as possible so that these capabilities can be built into the program architecture.