Inversion of Multi-Angle Radiation Measurements

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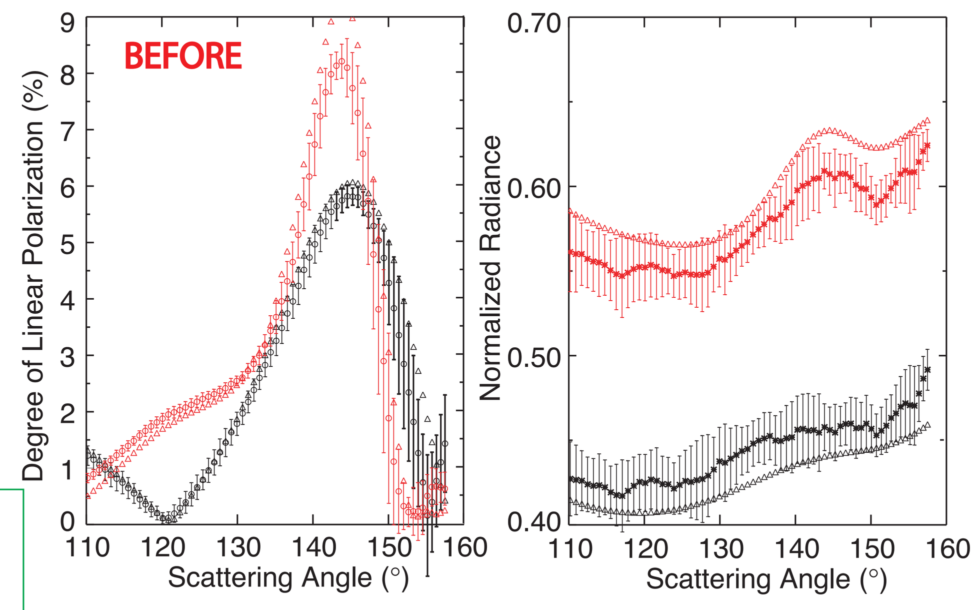
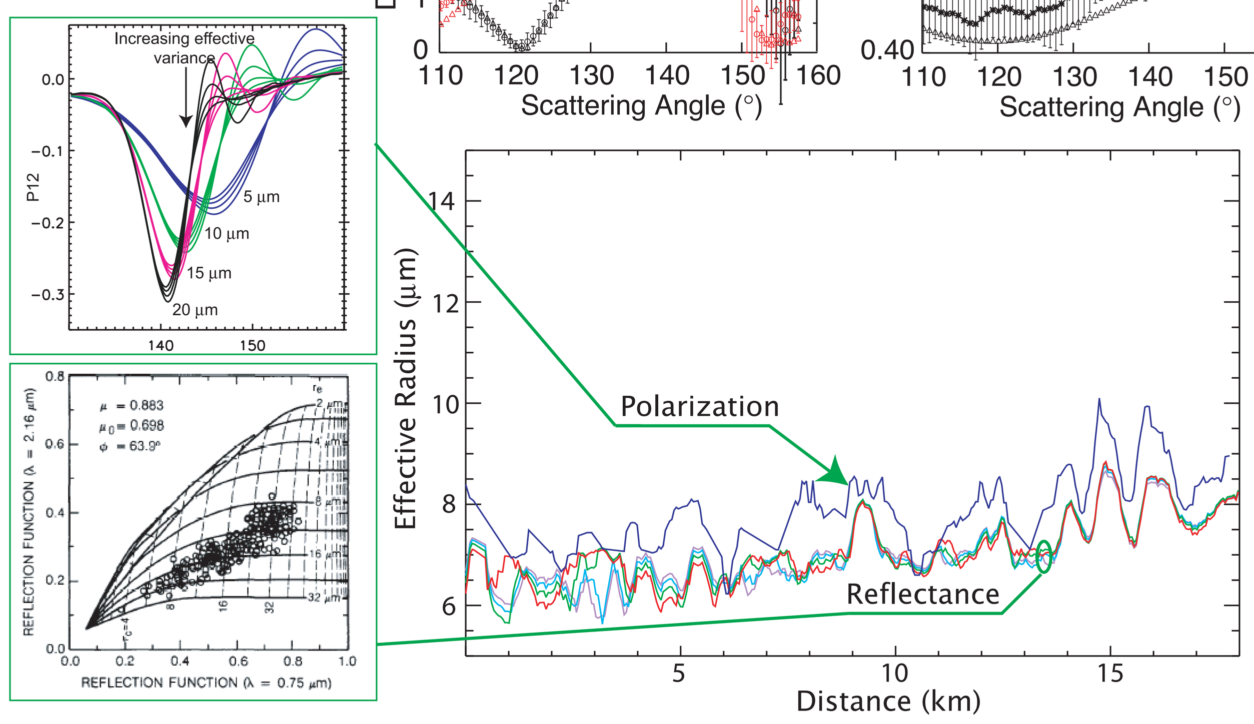
1. Motivation

Observations of the polarization and reflectance in bands at 865 nm (negligible liquid water absorption) and 2250 nm (strong liquid water absorption) cannot be fitted by a simple atmospheric model consisting of a homogeneous cloud with a single particle size.

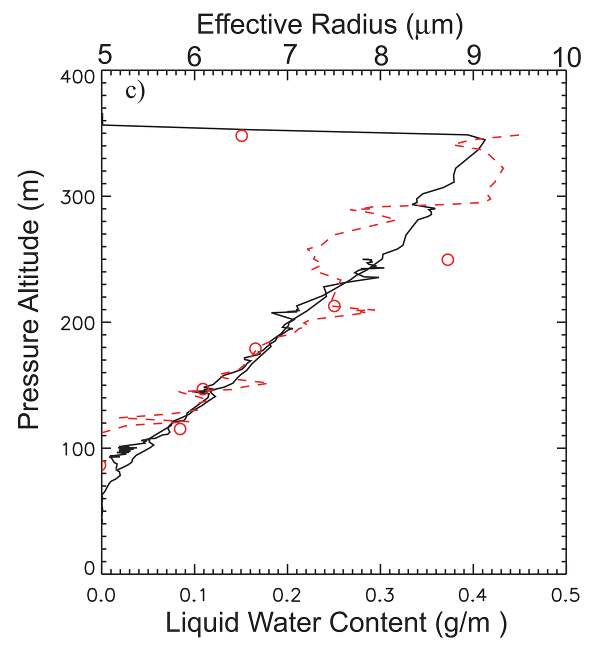


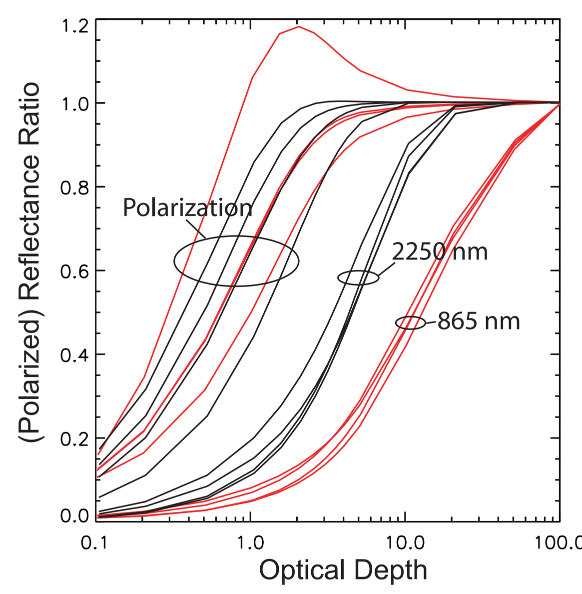
Triangles are model calculations for reff=5.9 µm veff=0.095 and an optical depth of 15.3. Circles with error bars are mean and standard error of observations for a 0.5 km data sample. Red is 865 nm and black is 2250 nm.

If we use the polarization (polarized reflectance) measurements and the reflectance measurements to retrieve cloud particle size independently we find that it is frequently (although not always) the case that the size retrieved with the polarization measurements is larger than the size retrieved using the reflectance measurements.



When we look at the in situ observations of particle size within the cloud on days when the polarization and reflectance size retrievals differ we find that there is a strong vertical gradient in particle size. If we then take a simple minded look at the optical depths that contribute to the observed (polarized) reflectance we find that the polarized reflectance measurements respond to an optical depth of 3 while the reflectance measurements responds down to an optical depth of 20 (2250 nm).





The question then is whether we can provide a practical method for retrieving the particle sizes in a vertically inhomogeneous cloud that is consistent with all the multi-angle, multi-spectral polarization and reflectance measurements that are available.

2. Perturbation Theory

The equation of transfer for polarized light can be written in operator form as **LI**= where the transport operator **L** is

$$\mathbf{L} = \mathbf{s}_1.\nabla_1 + \sigma_{ext}(\mathbf{r}_1)N(\mathbf{r}_1)\int_{4\pi}\delta(\mathbf{s}_1.\mathbf{s}_1') - \varpi\frac{\mathbf{P}(\mathbf{s}_1.\mathbf{s}_1')}{4\pi}\Sigma d\Omega_1'$$

I is the Stokes vector and **S** is the source term. The equation for the Green's function of this operator can formally be expressed as **LG**=δ(1,2) where δ(1,2) is a Dirac delta function in both space and angle variables. If we can determine the Green's function then the Stokes vector can be evaluated from the expression **I=GS**. If we now perturb the transport operator by altering the single scattering properties of the particles present, or the number density of scatterers and/or absorbers then the effects of this perturbation on the Stokes vector can be expressed as **(L+ΔL)I'=S**. The perturbed Stokes vector can be expressed as the sum of its unperturbed part **I**, and a perturbation **ΔI**. If we now neglect terms that are second order in the perturbation (i.e. the term **ΔLΔI**) we obtain the final result for the effects of perturbing the radiative transfer equation on the observed Stokes vector

$$\mathbf{I}'=\mathbf{I}-\mathbf{G}(0,\tau)\Delta\mathbf{L}(\tau)\mathbf{G}(\tau,0)\mathbf{S}$$

where the optical depth variables indicate that this expression is specific to reflection and the two Green's functions are different. One [**G**(τ,0)] allows the diffuse (and direct) radiation within the layer to be evaluated while the other [**G**(0,τ)] is the (Mueller matrix generalization of the) 'escape' function introduced by Twomey (1979).

3. Calculating Green's Functions

So, what are the Green's functions in terms of the more usual quantities that are calculated by radiative transfer codes?

$$\mathbf{G}(\tau,0;\mu,\mu';\varphi-\varphi')=\mathbf{U}(\tau;\mu,\mu';\varphi-\varphi') \;;\ \mu>0,\ \mu'<0 \quad \mathbf{G}(\tau,0;\mu,\mu';\varphi-\varphi')=\mathbf{D}(\tau;\mu,\mu';\varphi-\varphi') + \exp(\tau/\mu)\delta(\mu-\mu')\delta(\varphi-\varphi') \;;\ \mu<0,\ \mu'<0$$

$$\mathbf{G}(0,\tau;\mu,\mu';\varphi-\varphi')=\mathbf{U}_+(\tau;\mu,\mu';\varphi-\varphi') \;;\ \mu>0,\ \mu'<0 \qquad \mathbf{G}(\tau,0;\mu,\mu';\varphi-\varphi')=\mathbf{D}_+(\tau;\mu,\mu';\varphi-\varphi') + \exp(\tau/\mu)\delta(\mu-\mu')\delta(\varphi-\varphi') \;;\ \mu>0,\ \mu'>0$$

In these equations **D** and **U** are the usual downwelling and upwelling matrices and $\mathbf{D}_+$ and $\mathbf{U}_+$ are the (adjoint) downwelling and upwelling radiation fields that would result from a source being placed in the direction of observation.

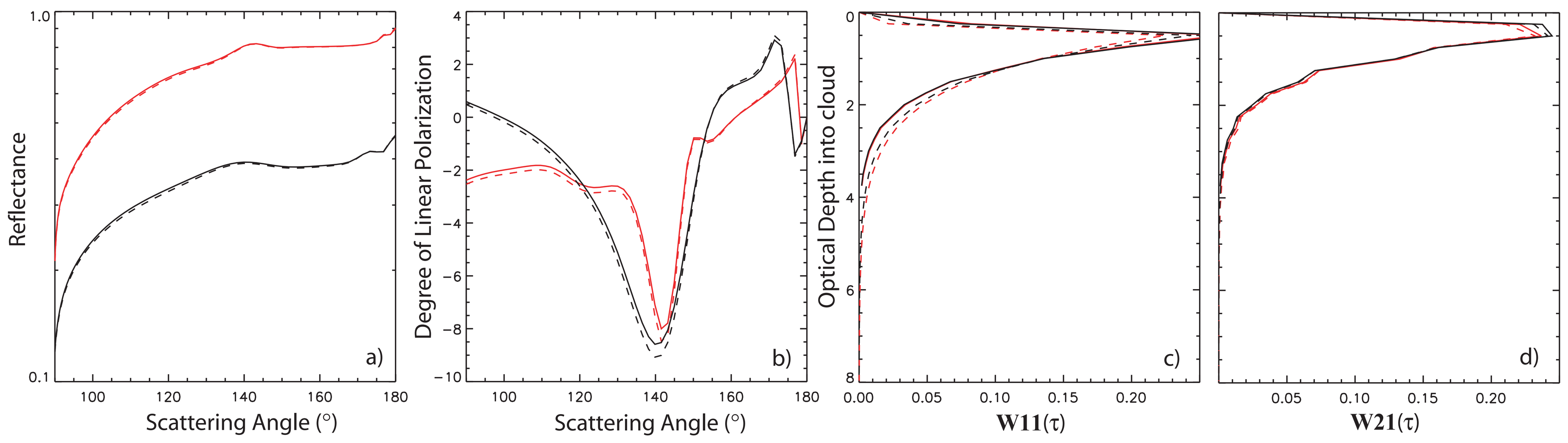
$$\mathbf{D}^+(\mu,\mu',\varphi-\varphi')=\mathbf{q}_4\mathbf{D}^T(\mu',\mu,\varphi-\varphi')\mathbf{q}_4 \qquad \mathbf{U}^+(\mu,\mu',\varphi-\varphi')=\mathbf{q}_4\mathbf{U}^T(\mu',\mu,\varphi-\varphi')\mathbf{q}_4 \qquad \mathbf{q}_4=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$$

4. Practical Implementation

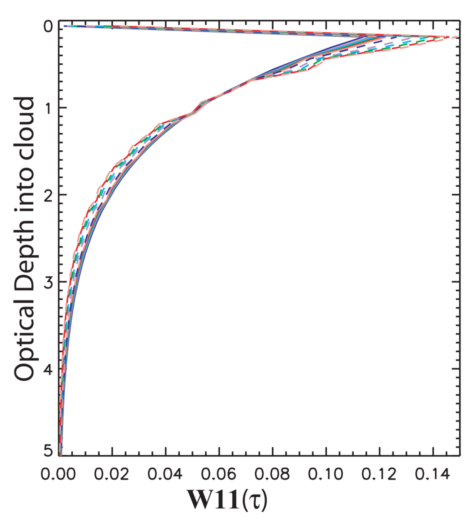
Discrete ordinates methods provide internal fields but for multi-angle observations calculations must be made for pseudo sources at all the observation angles in order to calculate the effect of perturbations. A method for the direct calculation of scalar (intensity only) Green's functions has been developed (Benedetti et al. 2000), but its generalization to realistic vertically inhomogeneous atmospheres and the inclusion of polarization (vector radiative transfer) is not trivial. However, doubling/adding calculations provide all the required angular informations and fast and accurate vector adding/doubling codes are readily available. So, if doubling/adding can be used to rapidly calculate the internal radiation field this approach will allow us to calculate the required Green's functions. In fact the downwelling and upwelling fields are required in the calculation of reflection and transmission matrices in the doubling/adding method and it is sufficient to save the last N downwelling and upwelling radiation fields from the doubling calculation of a layer in order to be able to calculate the internal field at 2^N-1 levels within the layer. Only simple multiplications of pairs of matrices are required in this calculation. The calculation of internal downwelling and upwelling fields as the atmosphere is added up is then quite straightforward (de Haan et al. 1984).

5. Weighting and Contribution Functions

The Green's functions can be used to calculate the effects of perturbations on the observed radiation field and also to examine the vertical weighting within a scattering medium that contributes to the observed field. The contribution function **C(τ)=G(0,τ)S(τ)** has been introduced by Benedetti et al. (2000). This function has the useful property that it satisfies the equation $\mathbf{R}=\int \mathbf{C}(\tau)d\tau$ which provides a useful check on the accuracy of the Green's function calculation. It also means that a natural definition of a weighting function is **W(τ)=C(τ)/R**.

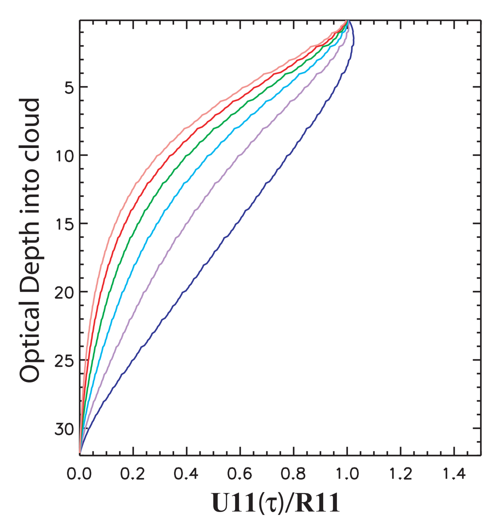


Figures a) and b) show direct (solid) and contribution function (dashed) calculations of reflectance and the degree of linear polarization from a cloud with reff=10 µm, veff=0.1 an optical depth of 32 and the sun overhead. Figures c) and d) show weighting function calculations for the Mueller matrix 11 and 21 elements respectively. Solid lines are for an optical depth of 4 and dashed lines are for an optical depth of 32. Lines in red are for 865 nm and in black are for 2250 nm.



The contribution (and weighting) functions can be a little misleading. They tell you what fraction of light that experienced its first scattering at an optical depth of τ returned to the surface. For strongly forward scattering media like clouds this is not the same as the contribution that light that reaches an optical depth τ makes.

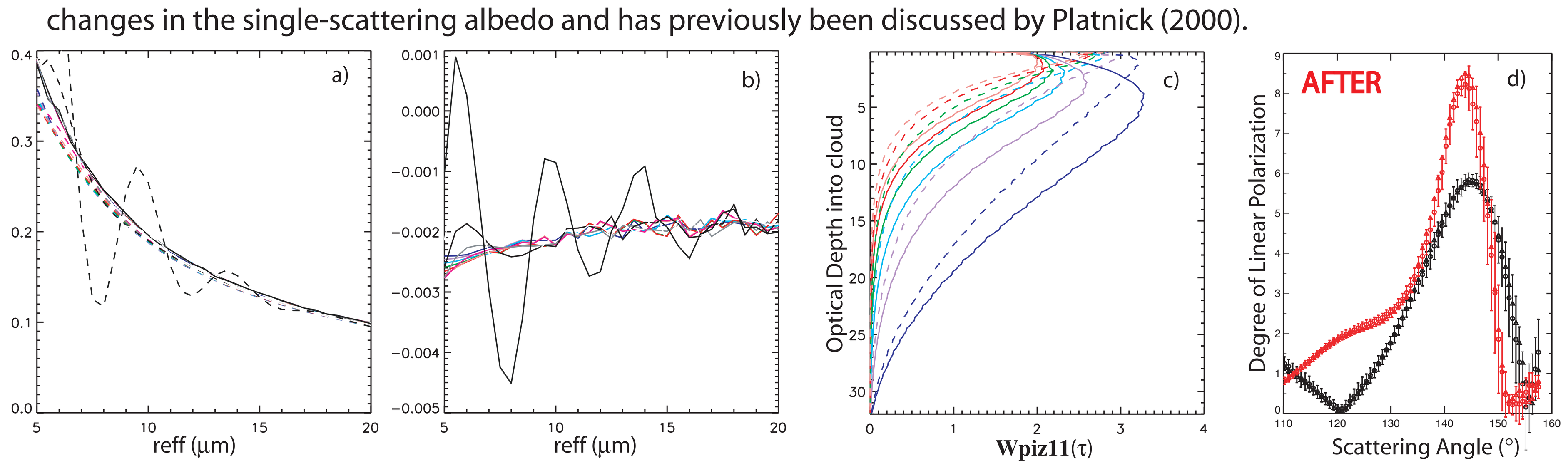
Comparing the two figures it is clear that radiation penetrates far deeper into the cloud than the weighting function suggests.



A more informative weighting function can be obtained by using the perturbation approach outlined in section 2. The relative perturbation in reflectance can be written as

$$\frac{\Delta\mathbf{R}}{\mathbf{R}}=\boxed{\int \Delta reff(\tau)\frac{\partial\ln\varpi(\tau)}{\partial reff}\mathbf{Wpiz}(\tau)d\tau}^{\,1}-\boxed{\int\left[\Delta reff(\tau)\frac{\partial\ln\sigma_{ext}}{\partial reff}+\Delta\ln N(\tau)\right]\mathbf{Wext}(\tau)d\tau}^{\,2}$$

where **Wext(τ)=G(0,τ)[I-P(τ)]G(τ,0)/R** and **Wpiz(τ)=G(0,τ)P(τ)G(τ,0)/R** are the new weighting functions. One (**Wext**) gives the vertical weighting of perturbations associate with changes in the extinction cross-section, number concentration of particles, while the other (**Wpiz**) gives the vertical weighting of perturbations associated with changes in the single-scattering albedo and has previously been discussed by Platnick (2000).



Figures a) and b) show the derivates of log extinction and log single scattering albedo as a function of effective radius. Dashed lines in a) are for 865 nm all other lines are for 2250 nm with veff varying from 0.01-0.2. Oscillations are apparent for veff=0.01 and veff=0.03. Figure c) shows the weighting function **Wpiz**. This function is of primary interest for remote sensing of particle size using absorbing and non-absorbing bands combined with polarization measurements. This is because if we have obtained a good reflectance retrieval and now wish to adjust the size deeper in the cloud to allow for the particle size determined at cloud top from the polarization measurements then term 2 above must be zero to maintain the reflectance fit at 865 nm (i.e any change caused by changing the effective radius must be compensated by changing the number concentration, or cloud thickness). The particle size decrease deep inside the cloud must therefore compensate the particle size increase at cloud top such that term 1 is zero in order to maintain the reflectance fit at 2250 nm. The final result is d).

6. Conclusions

Our need to reconcile models and measurements in an efficient manner that allows for the operational retrieval of particle sizes for a two layer cloud led us to develop a new method for calculating the Green's functions for radiative transfer. The method uses the fact that doubing/adding codes can be easily used to calculate internal radiation fields at arbitrarily high resolution. We have also determined that the adjoint downwelling and upwelling radiation fields are simply related to the usual downwelling and upwelling radiation fields so that the entire Green's function can be determined from a single calculation. The Green's functions have then been used to calculate the particles sizes in a two layer cloud that are consistent with both the reflectance and polarization measurements. This approach may be of use in other applications where adjoint calculations are used, particularly if multi-angle measurements are being analyzed.