Predicted vs observed results for different channel combinations, information content and operational error for multi-sensors SST retrievals

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Basic of Physical Inverse Model

- Forward model: $\mathrm{Y} = \mathrm{KX};\quad d\mathbf{Y} = \mathbf{K}d\mathbf{X}$
- Inverse: $d\mathbf{X} = \mathbf{K}^{-1}d\mathbf{Y}$ (measurement error)
- Lengendre (1805) developed Least Squares stochastically, but the deterministic form

$$\mathbf{X} = \mathbf{X}_{ig} + (\mathbf{K}^{\mathrm{T}}\mathbf{K})^{-1}\mathbf{K}^{\mathrm{T}}d\mathbf{Y}_{\delta};\quad dY_{\delta} = \mathbf{Y}_{\delta} - \mathbf{Y}_{ig}$$

Last 30~40 years: $\delta \mathrm{X} \leq \kappa\, \delta E;\ \kappa = \mathbf{cond}(\mathbf{K})$

- Two ways can be addressed:

$$dY_{\delta} - \delta Y = \mathbf{K}dX \quad then, \quad LS$$

$$\frac{\min}{\|\delta K\|, \|\delta Y\|, X} \{\|\delta \mathrm{K}\|^2 + \|\delta \mathrm{Y}\|^2\} \quad \text{subject to} \ \ (\mathrm{K}\text{-}\delta\mathrm{K})\,\mathrm{dX} = \mathrm{dY}_{\delta}\text{-}\delta\mathrm{Y}$$

Deterministic & Stochastic

Determinitic

$$\mathbf{X}_{rg} = \mathbf{X}_{ig} + (\mathbf{K}^{T}\mathbf{K} + \lambda\mathbf{I})^{-1}\mathbf{K}^{T} d\mathbf{Y}_{\delta}$$
$$= X_{ig} + K_{ps}^{inv}\, dY_{\delta}$$

TLS: $[\mathbf{u}\ \sigma\ \mathbf{v}] = [\mathbf{K}\quad d\mathbf{Y}_{\delta}]$

MTLS:

$$\lambda = (2\log(\kappa) / \left\| d\mathbf{Y}_{\delta} \right\|^{2})\, \sigma_{end}^{2}$$

Total Error: $\left\| \mathbf{X}_{true} - \mathbf{X}_{mtls} \right\|$

$$\left\| (\mathbf{K}_{ps}^{inv}\ \mathbf{K} - \mathbf{I})\mathbf{X}_{true} \right\| + \left\| \mathbf{K}_{ps}^{inv}(d\mathbf{Y}_{\delta} - \mathbf{K}\mathbf{x}_{mtls}) \right\|$$

Model Resolution Matrix:

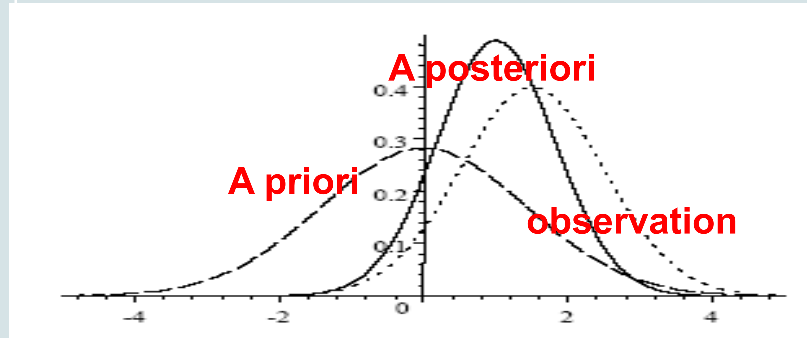
$$\mathbf{M}_{rm} = \left\{ (\mathbf{K}^{T}\mathbf{K} + \lambda\mathbf{R})^{-1}\mathbf{K}^{T} \right\} \mathbf{K}$$

Degree freedom in Retrieval:

$$DFR_{nor} = trace(\mathbf{M}_{rm}) / \min(m,n)$$

Stochastic/Probabilistic

$$\mathbf{X}_{rtv} = \mathbf{X}_{ig} + (\mathbf{K}^{T}\delta\mathbf{Y}^{-2}\mathbf{K} + d\mathbf{X}^{-2})^{-1}\mathbf{K}^{T}\delta\mathbf{Y}^{-2} d\mathbf{Y}_{\delta}$$



OEM: A set of measurement

$$\mathbf{X}_{oem} = \mathbf{X}_{ap} + (\mathbf{K}^{T}\mathbf{S}_{e}^{-1}\mathbf{K} + \mathbf{S}_{a}^{-1})^{-1}\mathbf{K}^{T}\mathbf{S}_{e}^{-1} d\mathbf{Y}_{\delta}$$

Chi-Square test:

$$\chi_{resd} = \mathbf{K}\mathbf{X}_{oem} - d\mathbf{Y}_{\delta}$$
$$\chi = \chi_{resd}^{T} (\mathbf{S}_{e}(\mathbf{K}^{T}\mathbf{S}_{a}\mathbf{K} + \mathbf{S}_{e})^{-1}\mathbf{S}_{e})^{-1} \chi_{resd}$$

Averaging Kernel:

$$\mathbf{A} = \left\{ (\mathbf{K}^{T}\ \mathbf{S}_{e}^{-1}\mathbf{K} + \mathbf{S}_{a}^{-1})^{-1}\mathbf{K}^{T}\mathbf{S}_{e}^{-1} \right\} \mathbf{K}$$
$$DFS_{nor} = trace(\mathbf{A}) / \min(m,n)$$

Characteristics of Inverse Methods

Elements	Deterministic	Stochastic
Measurement/s	True value + error	Expected value + uncertainty
Physical model	Necessary	Not always (e.g. regression)
Parameters	True value	Random variables
Inversion	Single pixel	A set of measurements
Validation(for a set of measurements)	RMSE= Systematic + Random	Bias (stability) + SD (uncertainty)
Names	Tikhonov, L-M, G-N, LS, TLS, RTLS, TSVD etc.	OE, M-L, 1d-var, Regression
EOS/Satellite inversion	A little known	Widely used

Information Content

Based on Shannon & Weaver (1949) information content study based on stochastic assumptions:

- Rodgers stated (p. 34-37, 2000): information of measurement is the changing of entropy of the state space before and after measurement and it is given for remote sensing radiative transfer inverse problem as:

$$H = S(p_1) - S(p_2)$$

- After simplification final form for information:
- $$H = -\frac{1}{2}\ln|I - A|$$ **For LS, A=I, H=0!**

Data and Forward model specifications

- Forward model using ver. CRTM2.1
- Monthly matchups pixel collocated data
- Buoy (coastal, Moore & drifters)
- Sensors: GOES13, MTSAT2, MODIS-A, VIIRS
- iQUAM quality control data
- Using GFS ancillary data (NRT operational)
- Night time scenarios
- CMIP5 climatology standard aerosol
- OEM error covariance: difficult in operation
- Cloud detection is major issue
- Bias for Skin-bulk, forward model and measurement

OEM error covariances

VIIRS (3.7 4.0 11 12): Ambiguities

0.12 0.04 0.03 0.03 (Boryana Efremova et al JGR 2014)

0.065 0.078 0.038 0.070 (JPSS ATBD 474 474-00048)

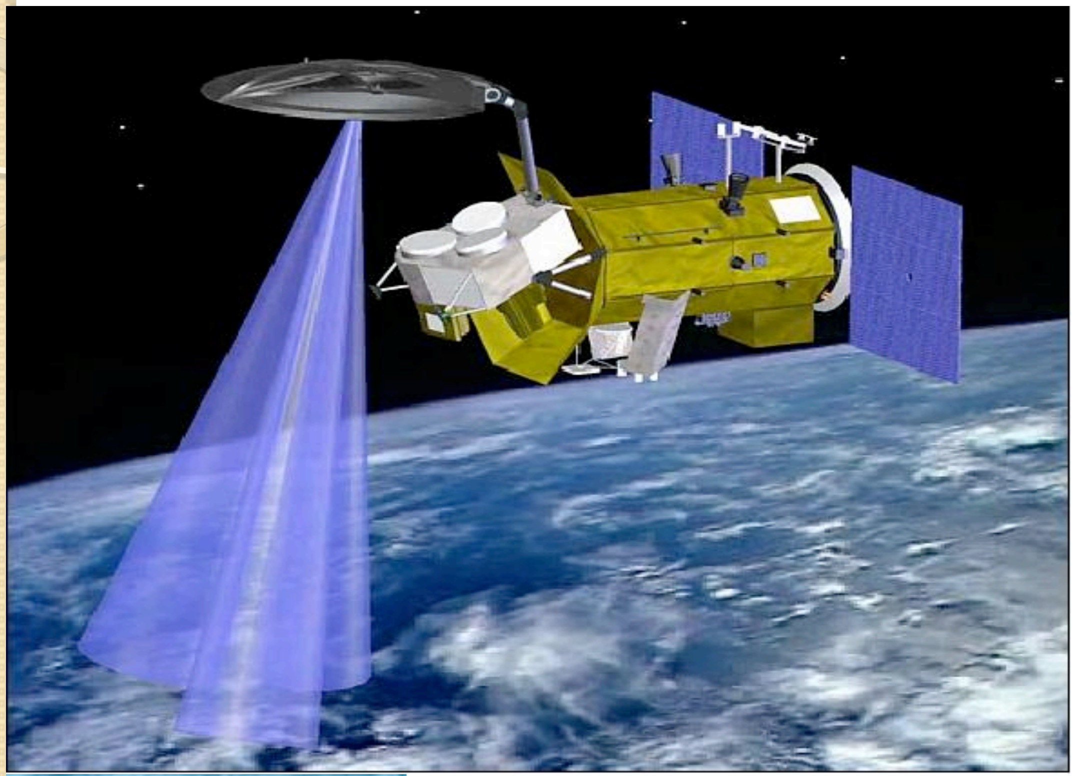
MTSAT2: http://www.wmo-sat.info/oscar/instruments

MODIS-A: Xiaoxiong Xiong IEEE TGRS,47, 2009

GOES13: NOAA Technical Report NESDIS 131

Fast forward model (CRTM2.1) error. It is very difficult to estimate correct forward model error. We assumed: ~0.2K near 4 μm channels (due to many absorbers in this region, which is considered in CRTM2.1) and ~0.1K for other channels.

Clear Sky Assumptions



Experimental Filter

$$rtv_{3.9} = (T_{3.9} - BT_{3.9}) / K_{3.9}$$

$$abs(SSTb - rtv3.9) < 1$$

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Systematic Errors for various model

1. Forward model biases (SRF, approximation RT equations, Parameterizations, profiles etc.
2. Instrument biases (calibration, recalibration, drifting etc.)
3. References biases (systematic skin-bulk error)

Experimental set up: Bias correction (BC_{ls}) is made based on the mean difference between the LS solution and SST_b. MTLS or TLS based algorithm minimizes the cost function using orthogonal LS, as compared to ordinary LS equally weights all measurement.. Thus MTLS bias correction is made:

$$BC_{mtls} = \frac{\sum_{i=1}^{m}\omega_i}{m \times \max(\omega_i)} BC_{ls}; \quad \omega = K_{sst}$$

Bias is an error. It generates from Models errors and should be objectively corrected at source.

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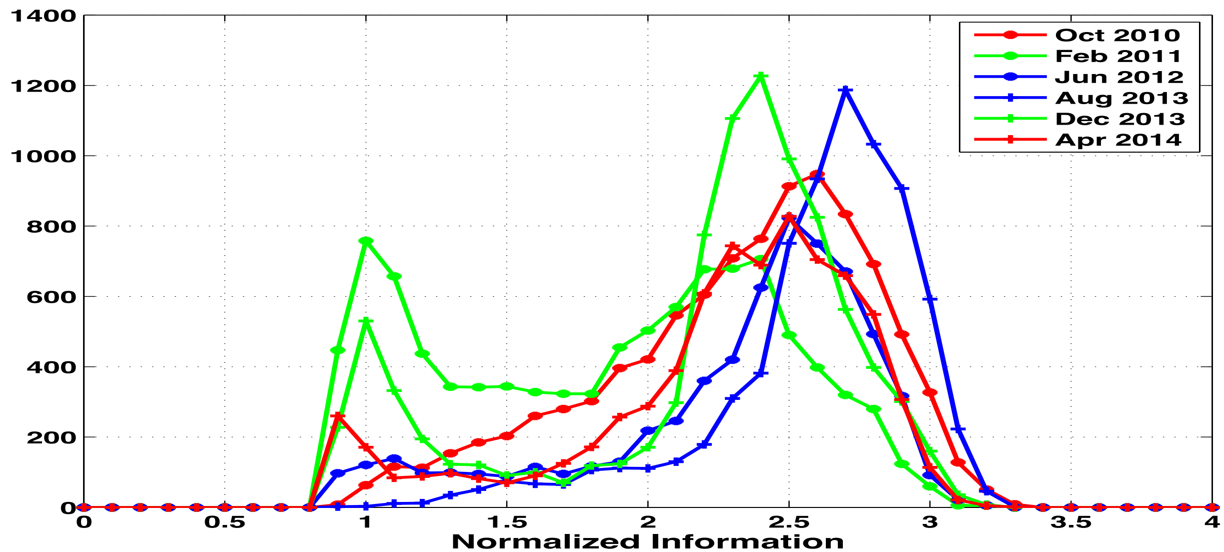
Normalized Information for SST retrieval from GOES13 using OEM

- NI=H/min(m,n)



❑ One measurement cannot produce more than one piece of information.

Degree of Freedom

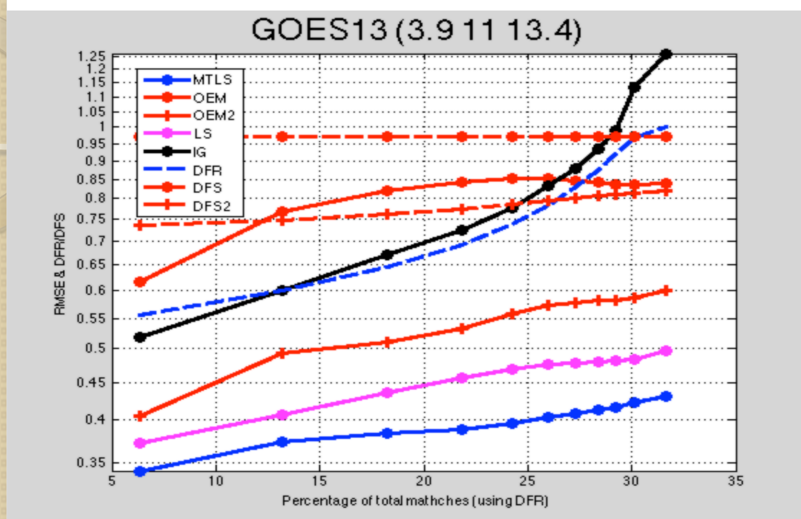
$$DFR_{nor} = trace(\mathbf{M}_{rm}) / \min(m,n)$$

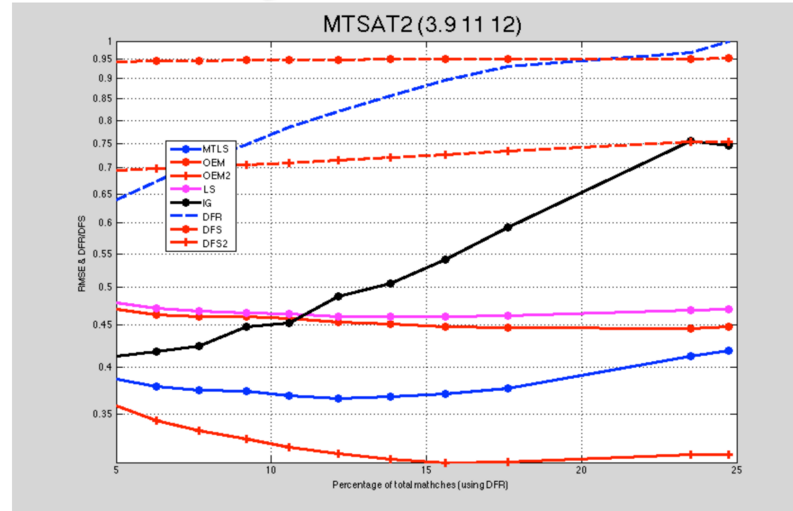
$$DFS_{nor} = trace(\mathbf{A}) / \min(m,n)$$

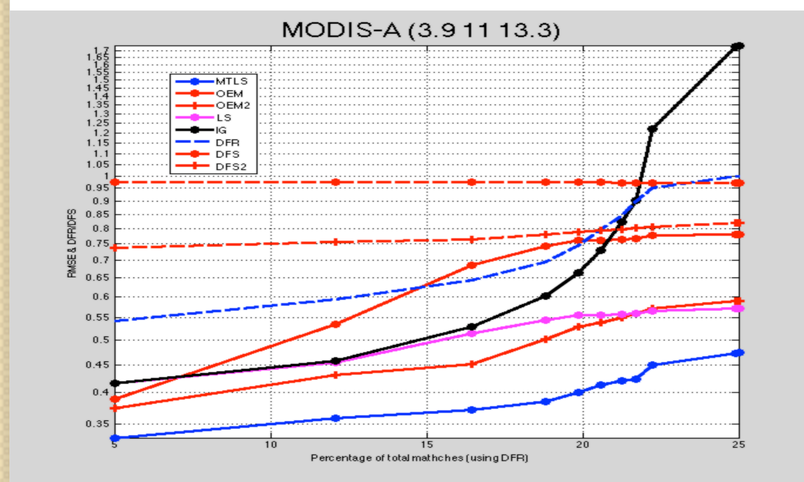
$$\mathbf{A} = \left\{(\mathbf{K}^{T}\,\mathbf{S}_{e}^{-1}\mathbf{K} + \mathbf{S}_{a}^{-1})^{-1}\mathbf{K}^{T}\mathbf{S}_{e}^{-1}\right\}\mathbf{K}; \quad \mathbf{M}_{rm} = \left\{(\mathbf{K}^{T}\mathbf{K} + \lambda\mathbf{R})^{-1}\mathbf{K}^{T}\right\}\mathbf{K}$$

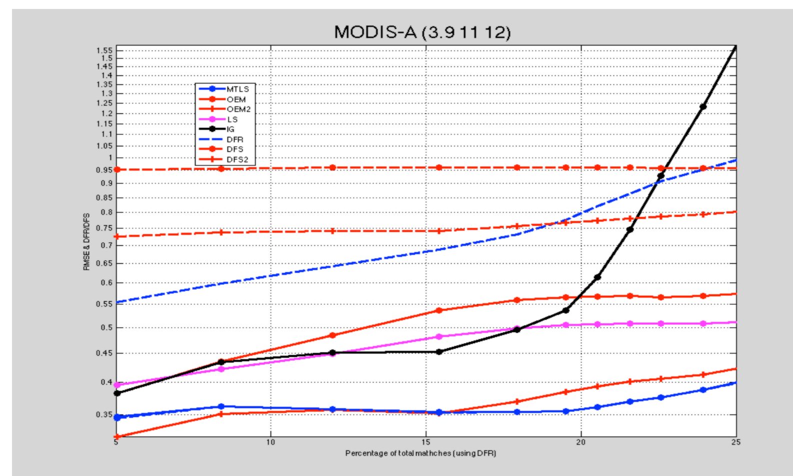
- **Normalized DFS/DFR of LS is one.**
- **Thus we add LS in comparison study of MTLS & OEM as a reference.**

DFS/DFR and Retrieval error using three sensors for the month of June 2014









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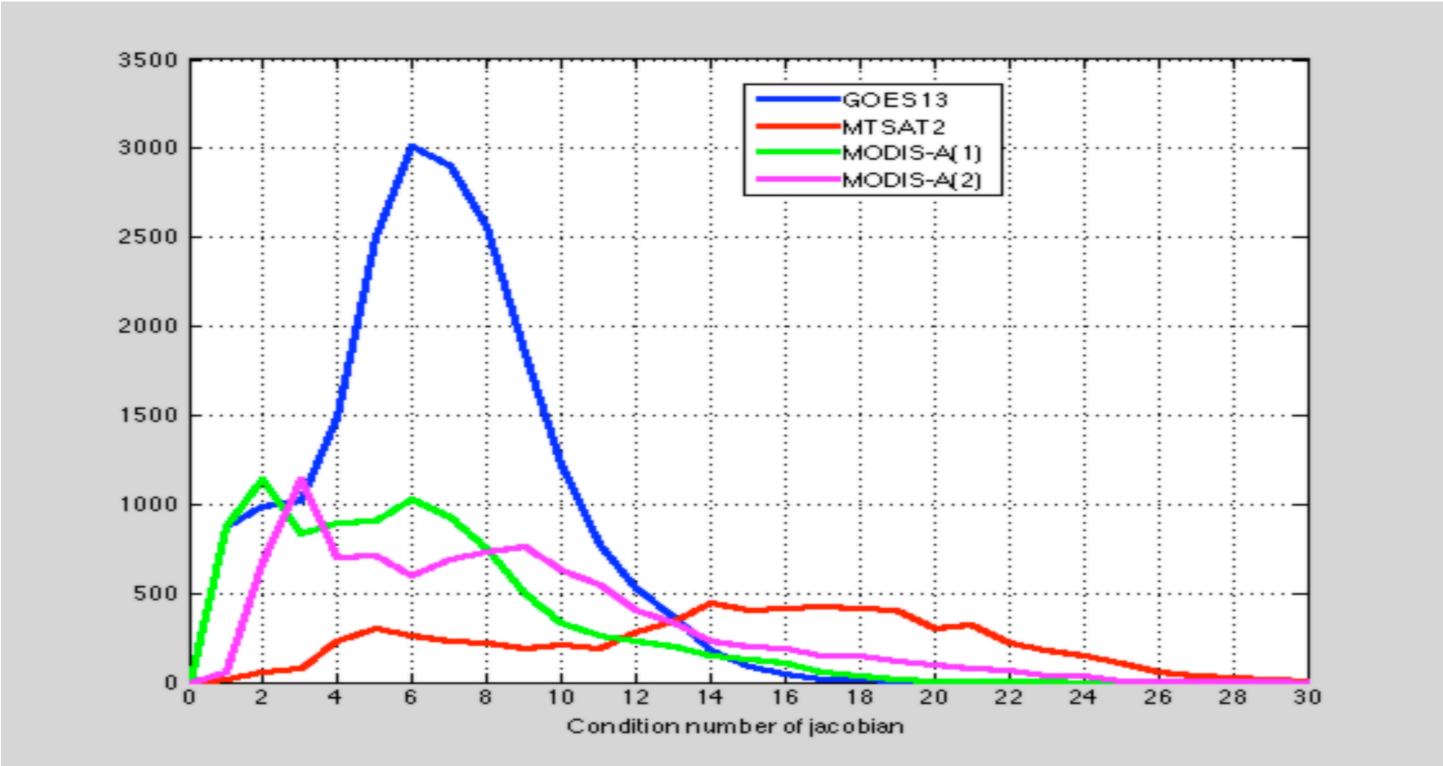
Distribution of Condition number



- Condition number of jacobian containing 13.4 μm hannel is lesser than the same of 12 μm channel.

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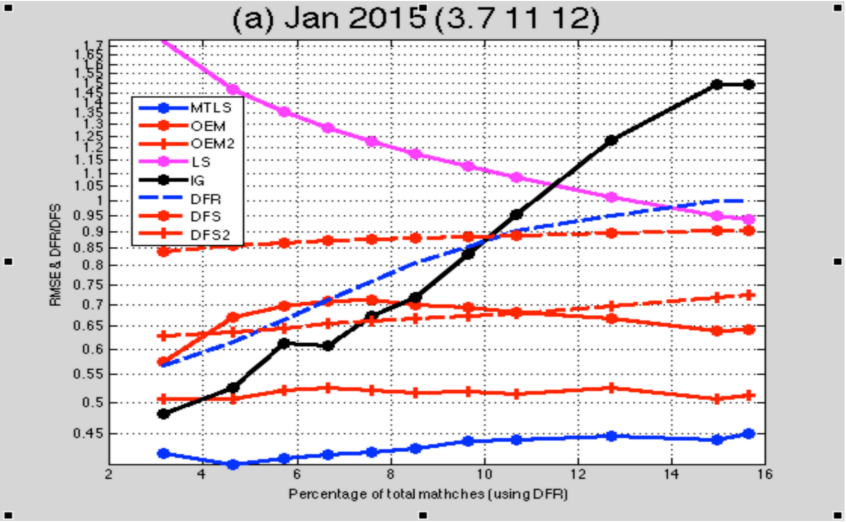
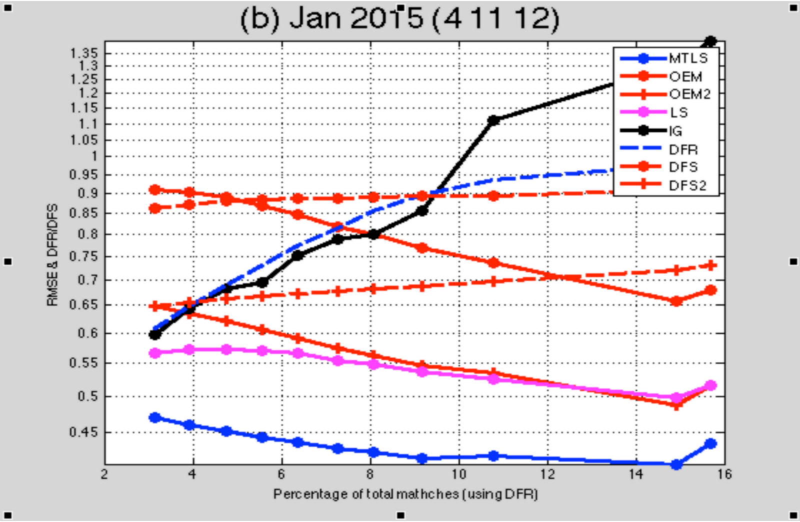
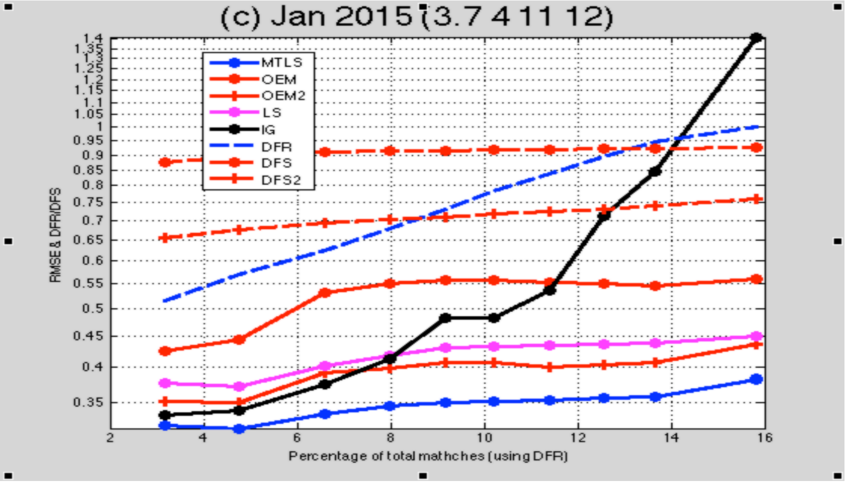
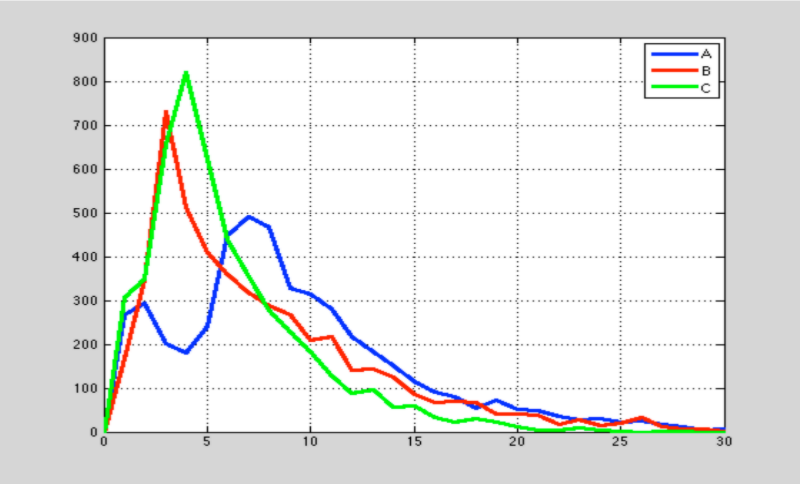
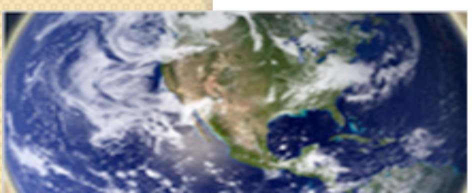
DFR/DFS of VIIRS for various channels combinations



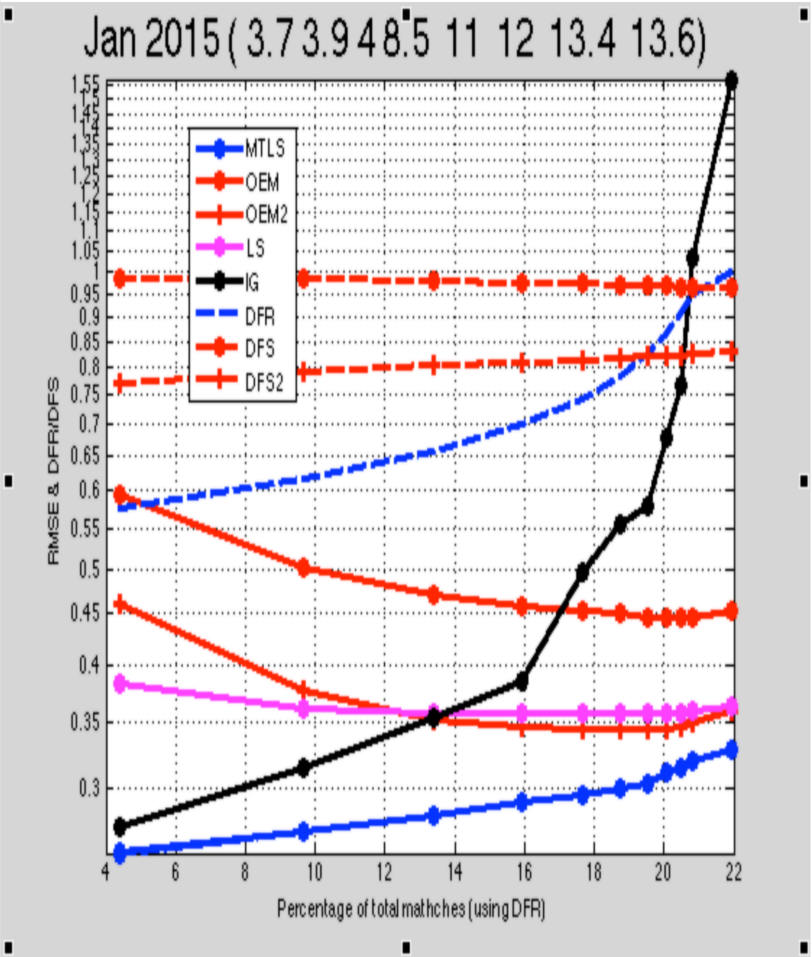


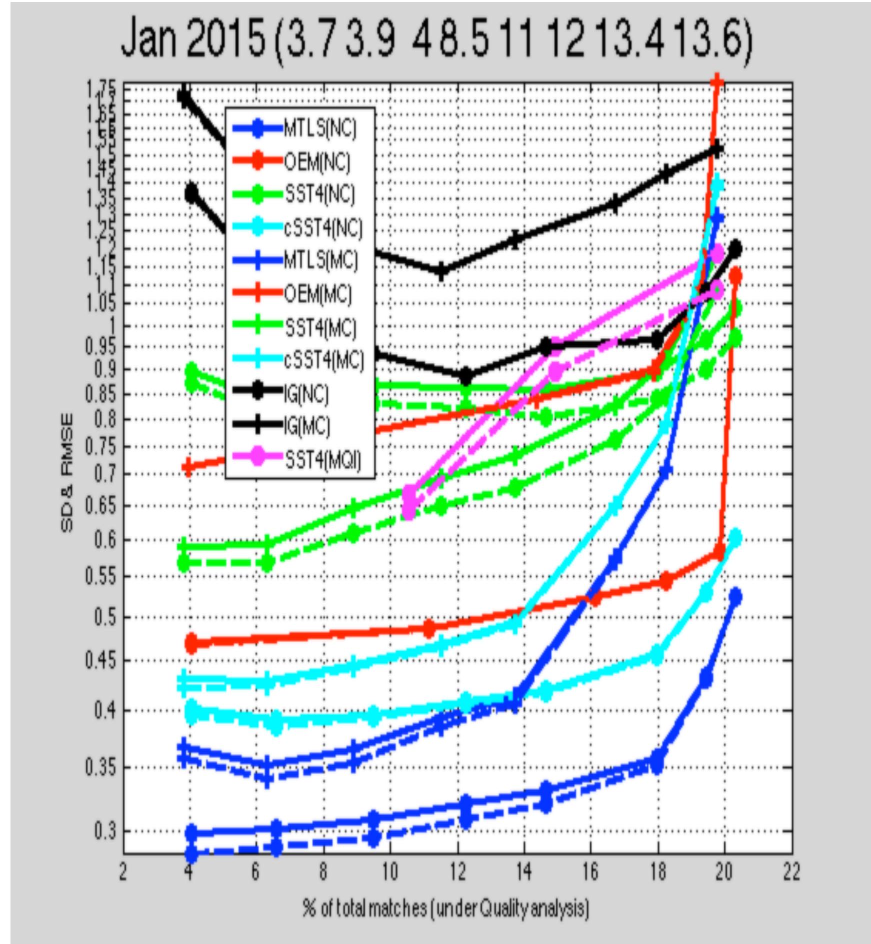






Results of MODIS-A for multichannels





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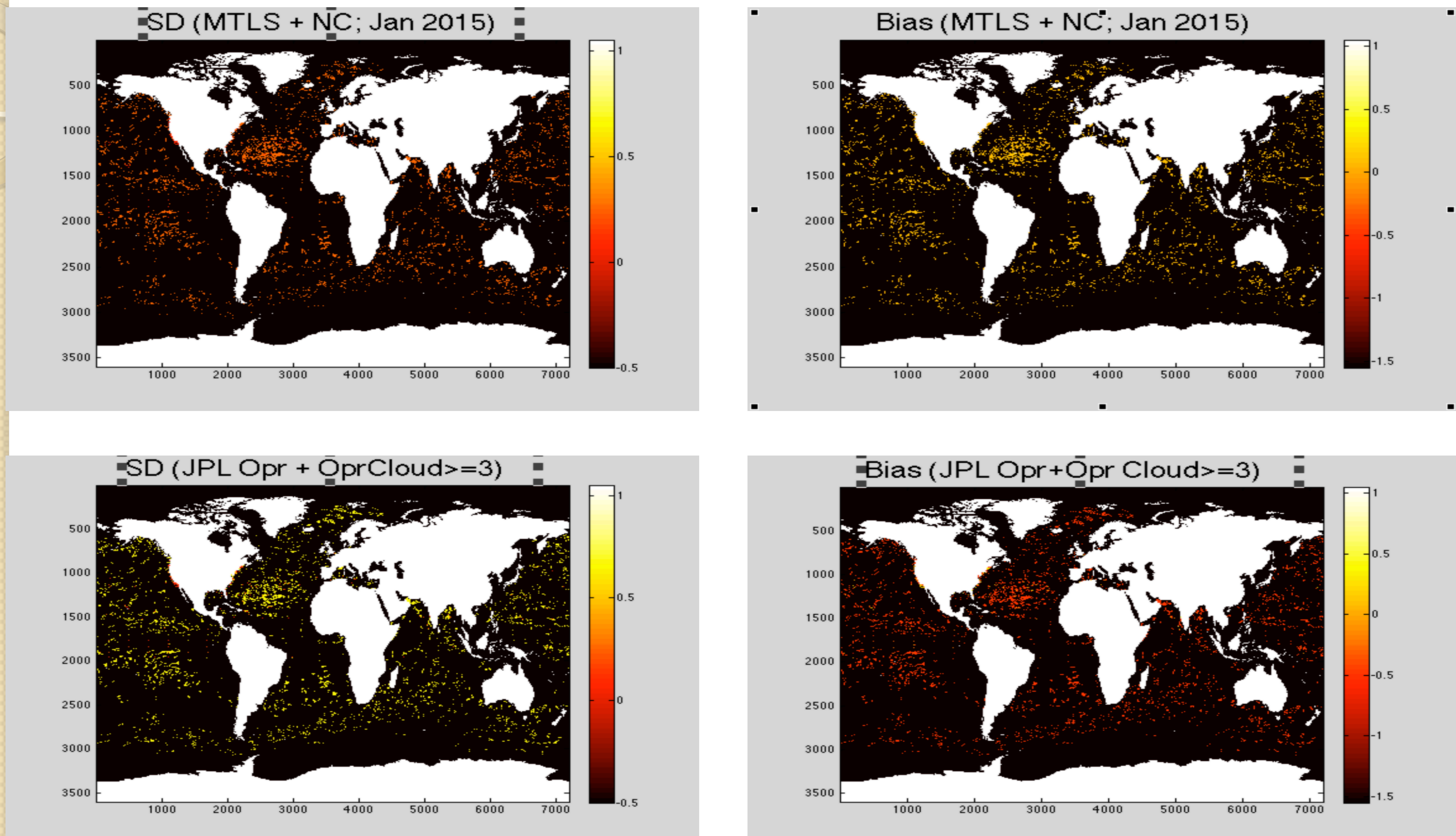
Validation Map for MODIS-A SST



Summary and conclusions

- Developmental history of inverse algorithms and sensitivity study.
- In our study, MTLS shows the best performance
- This study also shows that for majority of cases, OEM solutions contain higher error than that of a priori.
- Additionally, whether OEM outperforms LS or vice versa depends on the condition number of the problem in hand. (discussed theoretically at the beginning, and shown practically)
- Sensitivity study shows that: a low DFR/DFS does not necessarily mean a more accurate product. In other words, DFR alone is inadequate to characterize the true sensitivity.
- The success of MTLS is attributed to its data-driven regularization, i.e., when IG error is high, regularization is low and vice versa.

THANKS!